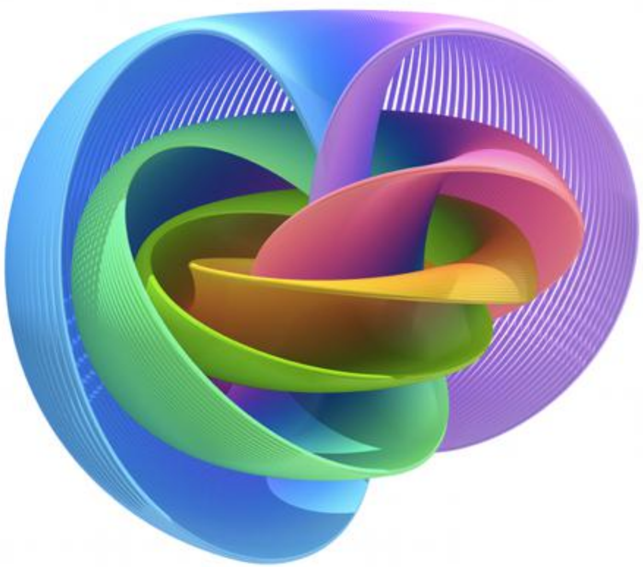
Materia Topólogica

M. Asorey

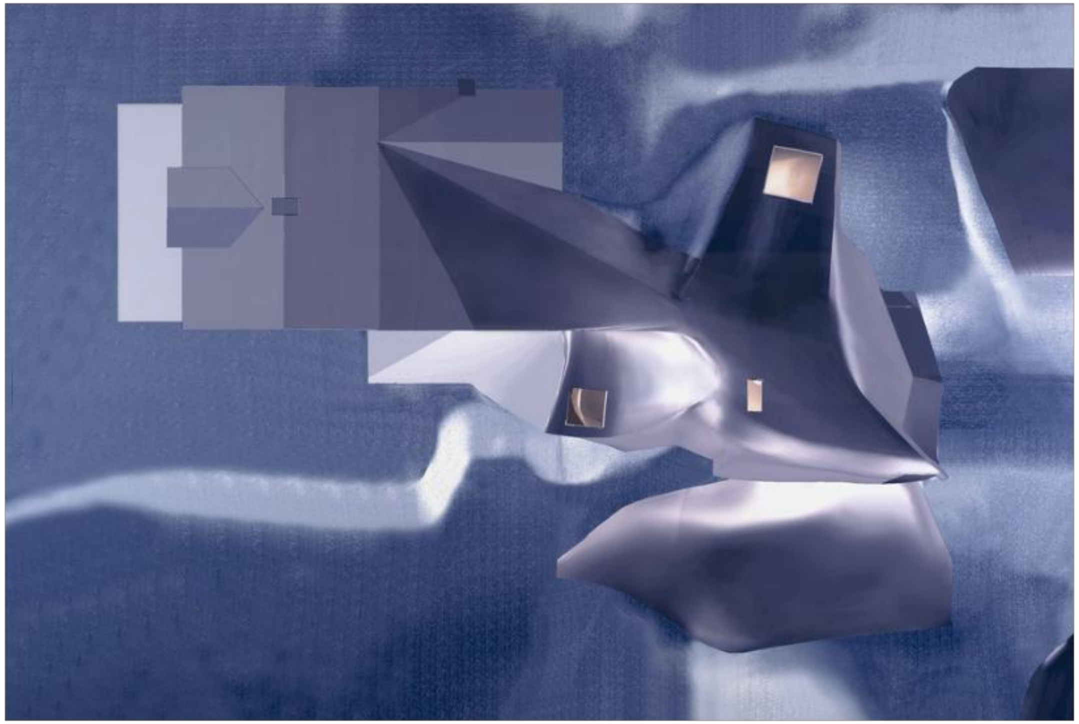


Arquitectura Topológica

- Architectural topology is the mutation of form, structure, context, and program into interwoven patterns and complex dynamics … topological 'space' differs from Cartesian space in that it imbricates temporal events-within form. Space then, is no longer a vacuum within which subjects and objects are contained, space is instead transformed into an interconnected, dense web of particularities and singularities better understood as substance or filled space (Stephen Perrela, U. Columbia, NY).

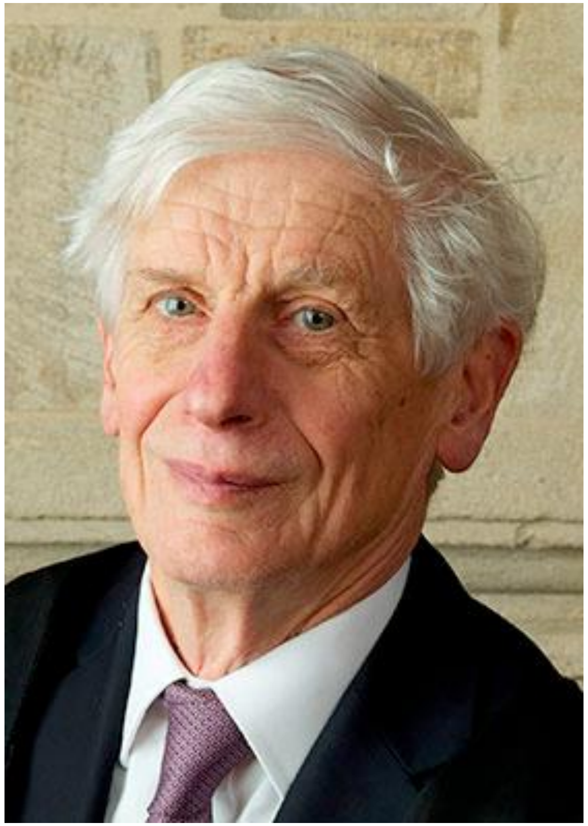
- Topology in architecture, involves measure and/or procedure of transformation of elements from one building to another (Scott Cohen, U. Harvard, MA).

Raybould House and Garden

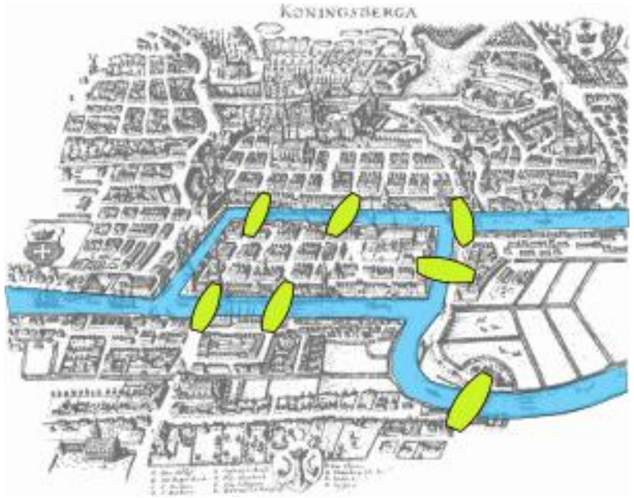


Guggenheim Building (Bilbao)

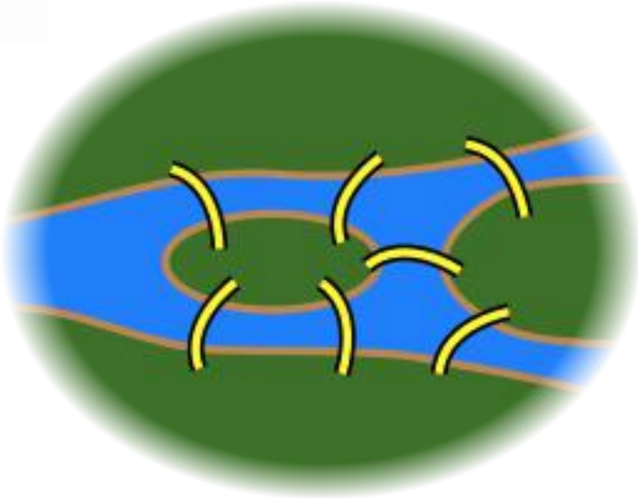
No tomarás el nombre topológico en vano



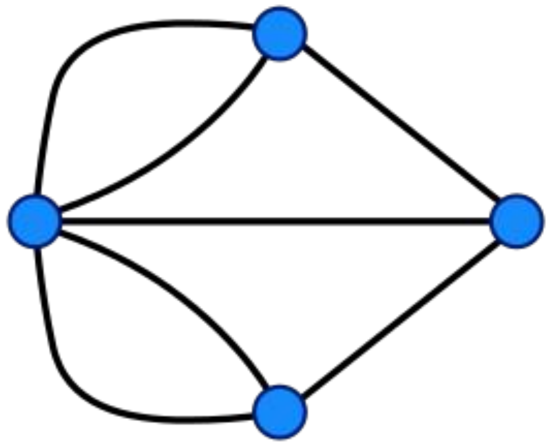
The Nobel Prize in Physics 2016 was divided, one half awarded to David J. Thouless, the other half jointly to F. Duncan M. Haldane and J. Michael Kosterlitz "for theoretical discoveries of topological phase transitions and topological phases of matter"



Leonard Euler

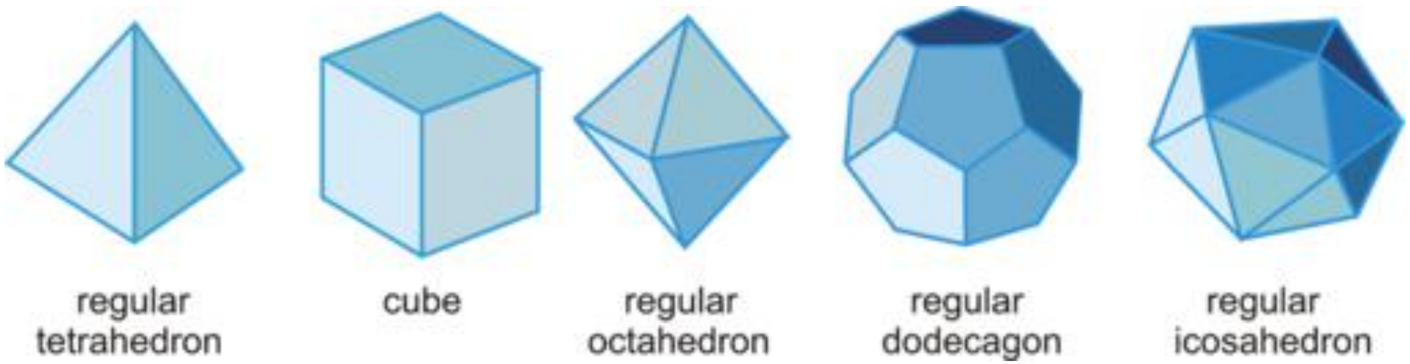


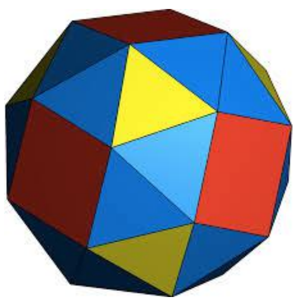
Problema de los puentes de Könisberg

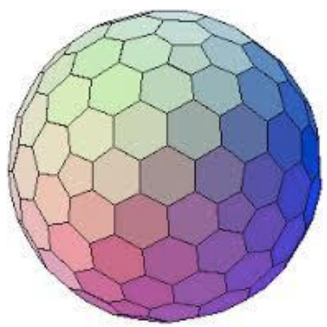


Teorema de Euler

$$C - A + V = \color{red}{2} \longleftarrow$$ Característica de Euler







$$\frac{1}{4\pi}\int_{S^2}\sqrt{g}R = 2$$

MATERIA TOPOLÓGICA

- Fases de la materia con parámetro de orden topológico
- Efecto Hall Cuántico
- Efecto Hall Cuántico Anómalo. Corrientes de borde
- Efecto Hall Cuántico de Spin
- Aislantes Topológicos
- Fermiones de Majorana
- Código Tórico de Kitaev
- Semimetales Topológicos. Fermiones de Weyl. Arcos de Fermi
- Semimetales con puntos nodales

Estabilidad por la robustez que proporciona topología

Berezinskii-Kosterlitz-Thouless phase transition

Coleman-Mermin-Wagner theorem:

No hay ruptura de simetría continua en 2D

Goldstone theorem:

Por cada simetría continua rota aparecen fluctuaciones de un boson sin masa próximas al vacío (no mass gap). Pero bosones libres sin masa no existen en 2D.

$$\langle 0|\phi(x)\phi(y)|0\rangle = -\frac{1}{2\pi}\log\frac{|x-y|}{a}$$

Berezinskii-Kosterlitz-Thouless phase transition

La no existencia ruptura de simetrías continuas no implica la ausencia se transiciones de fase en sistemas con simetrias continuas en 2D

Modelo sigma no lineal O(2)

$$\Phi(x) = e^{i\phi(x)}$$

$$\langle 0|\Phi(x)\Phi(y)|0\rangle = \frac{a^{2\pi}}{|x-y|^{2\pi}}$$

spin wave approximation, ...

Haldane gap and Haldane map

Heisenberg spin chain

$$H = -J \sum_{n=0}^{N} S_n \cdot S_{n+1}$$

$s = \frac{1}{2}$ Critical: no mass gap

$s = 1$ Non-critical: mass gap

Haldane conjecture is based on Haldane map

Haldane map maps Heisenberg spin chain into an O(3) sigma model

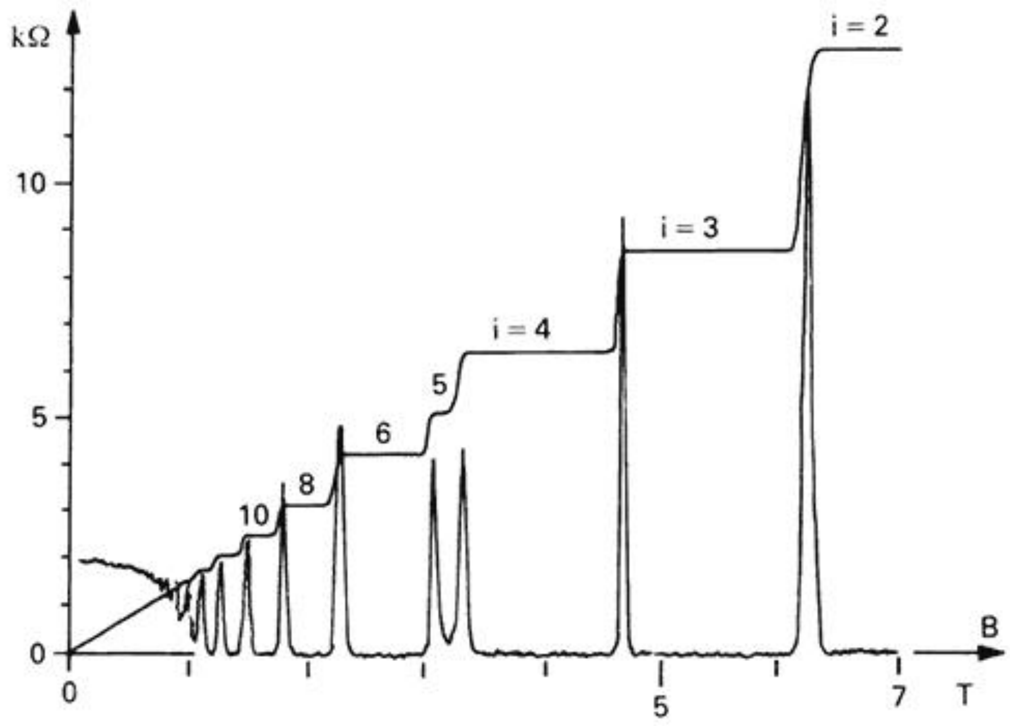
$$I = \frac{1}{2g^2} \int \partial^\mu \mathbf{n} \cdot \partial_\mu \mathbf{n} + \frac{k}{8\pi} \int \epsilon^{\mu\nu} \mathbf{n} \cdot \partial_\mu \mathbf{n} \times \partial_\nu \mathbf{n}$$

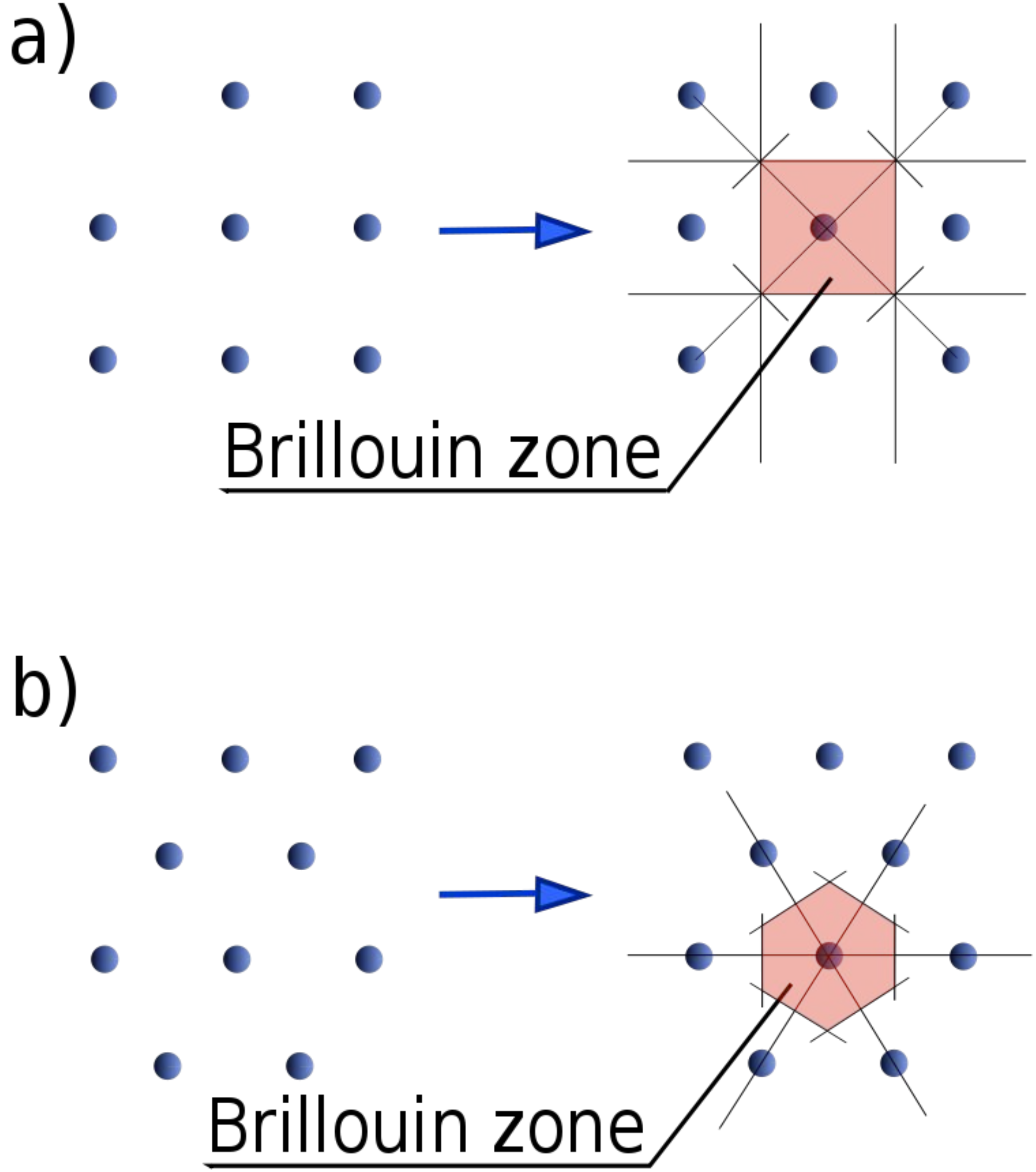
winding number

$$s = \frac{1}{2} \longrightarrow k = 1$$

$$s = 1 \longrightarrow k = 0$$

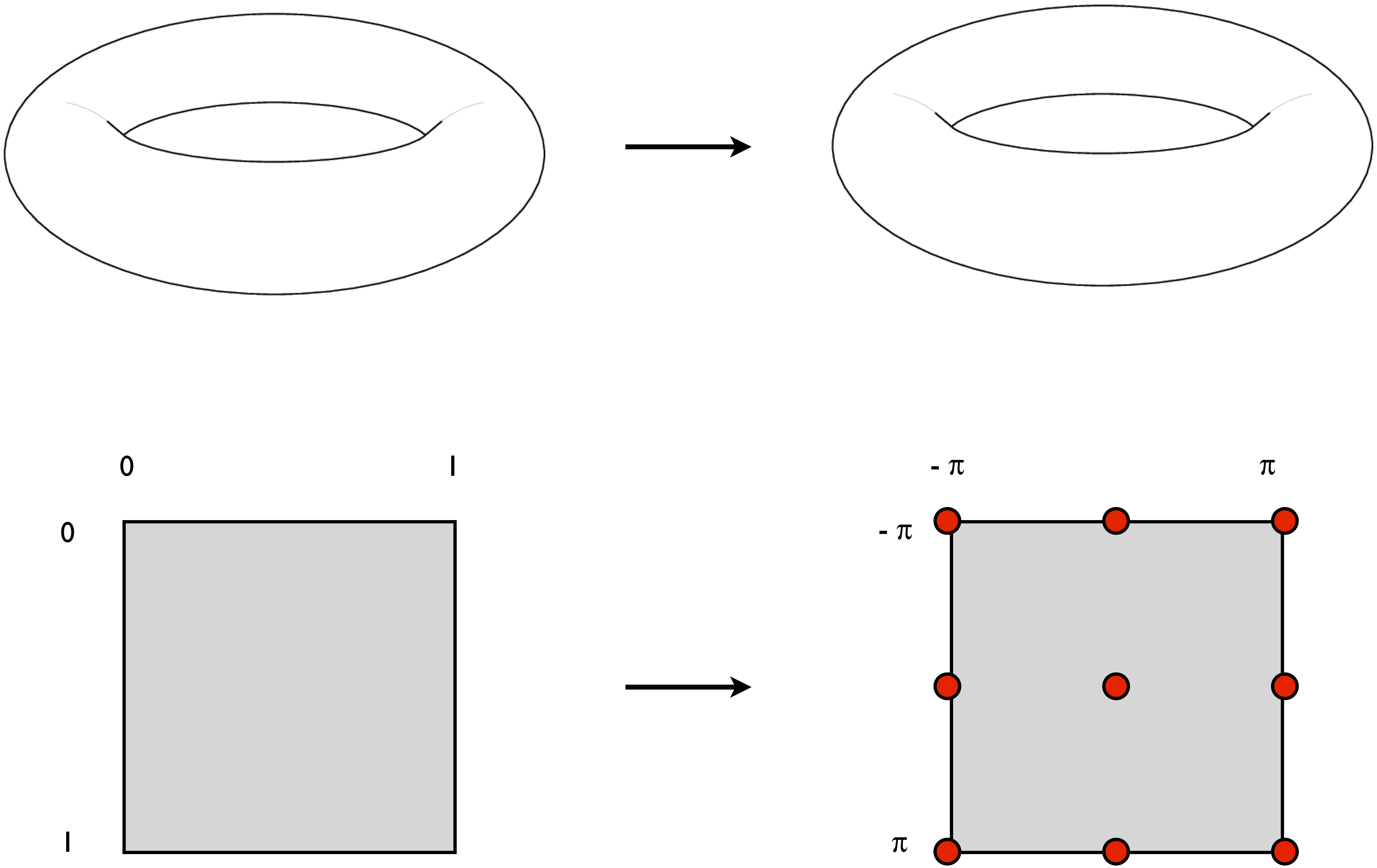
TKKN and Integer Quantum Hall effect




a)
Brillouin zone
b)
Brillouin zone

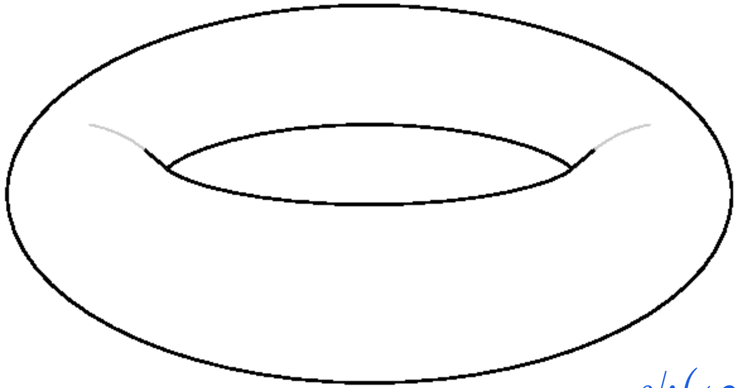
Real Space

Brillouin Zone



Kramers points

Hall Effect in 2D Torus $\mathbb{T}$



$$\frac{e}{2\pi}\int_T \mathbf{F} = k \in \mathbb{Z} \qquad k = eB/2\pi$$

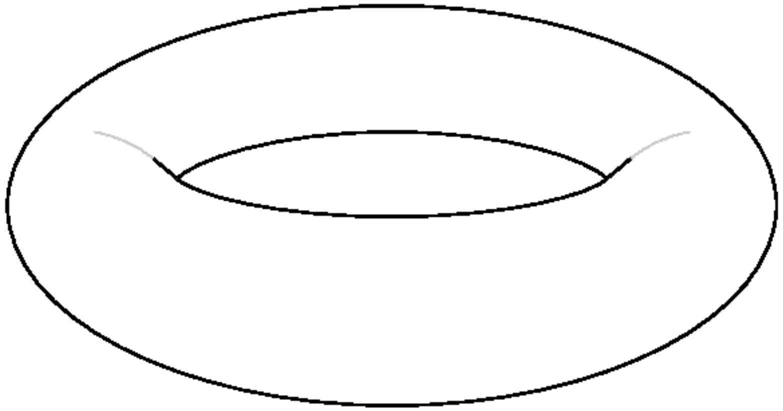
$$\begin{aligned}\psi(\varphi_1 + 2\pi, \varphi_2) &= e^{i\frac{k}{2}\varphi_2}\psi(\varphi_1 + 2\pi, \varphi_2)\\ \psi(\varphi_1, \varphi_2 + 2\pi) &= e^{-i\frac{k}{2}\varphi_1}\psi(\varphi_1, \varphi_2 + 2\pi).\end{aligned}$$

$$\mathbb{H} = \frac{1}{2m}\left[\left(\partial_1 + i\frac{B}{2}(\phi_2 + \epsilon_2)\right)^2 + \left(\partial_2 + i\frac{B}{2}(-\phi_1 - \epsilon_1)\right)^2\right]$$

Energy levels (degeneracy: $|k|$)

$$E_n = \frac{2\pi|k|}{m}\left(n + \frac{1}{2}\right)$$

Hall Effect in 2D Torus $\mathbb{T}$



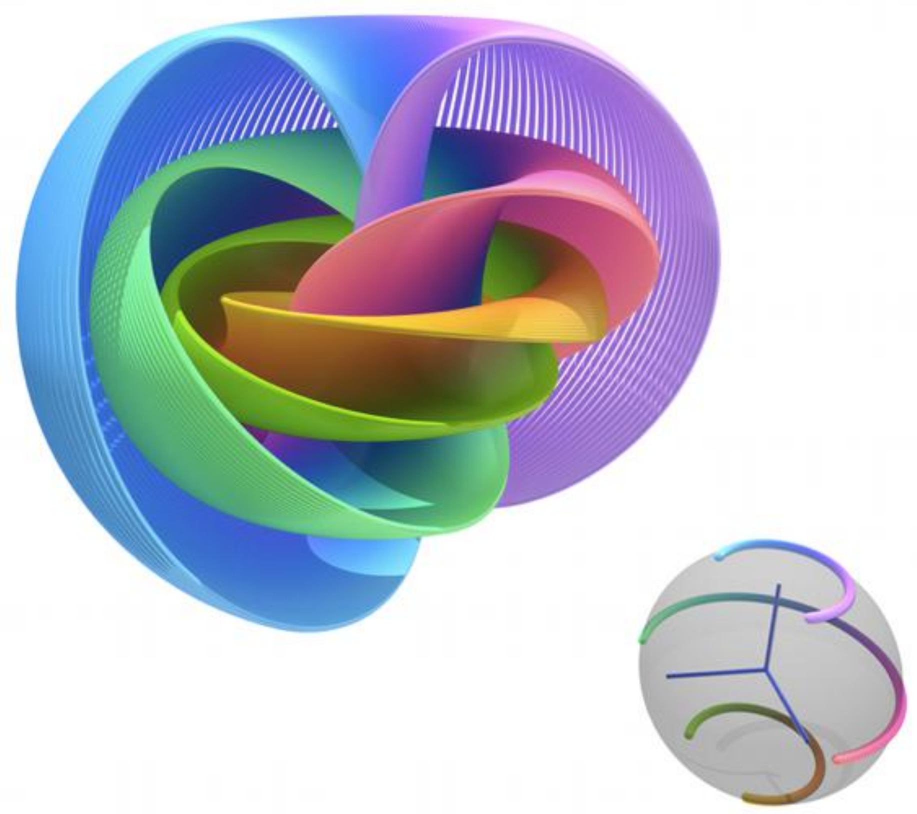
$$\frac{e}{2\pi}\int_T \mathbf{F} = k \in \mathbb{Z} \quad k = eB/2\pi$$

Ground State Eigenfunctions (degeneracy: $|k|$) :
Holomorphic sections of $E(T^2, \mathbb{C})$

$$\psi_0^l(\epsilon,\phi) = \frac{\mathrm{e}^{i\frac{k}{4\pi}(\phi_1+2\epsilon_1)\phi_2}}{(8\pi^4 k)^{\frac{1}{4}}} \sum_{m=-\infty}^{\infty} \mathrm{e}^{im\left(\phi_1+\epsilon_1+2\pi\frac{l}{k}\right)-\frac{1}{4\pi k}(2\pi m+k\phi_2+k\epsilon_2)^2}$$

$$l = 0, 1, 2\ldots, |k| - 1.$$

Torus $\longrightarrow$ Sphere S^2



TKKN and the Bloch bundle

The states with energies below the Fermi level define a vector bundle over the Brillouin zone torus.

In this bundle there are gauge fields defined by the Berry phases of the different states

$$\mathcal{A}^{l,l'}_{j\,n}(\epsilon) = -i \int_{\mathbb{T}^2} {\psi^{l}_{n}}^{*} \partial_{\epsilon_j} \psi^{l'}_{n}$$

$$\mathcal{F}^{l,l'}_{n}(\epsilon) = \partial_{\epsilon_1} \mathcal{A}^{l,l'}_{2\,n} - \partial_{\epsilon_2} \mathcal{A}^{l,l'}_{1\,n}$$

TKKN and the Bloch bundle

First Chern class of Bloch bundle

$$C_1 = -\frac{i}{4\pi} \sum_{n=0}^{\nu} \sum_{l=0}^{|k|-1} \int_{\widehat{\mathbb{T}}^2} \mathcal{F}_n^{l,l} = \nu$$

TKKN formula:

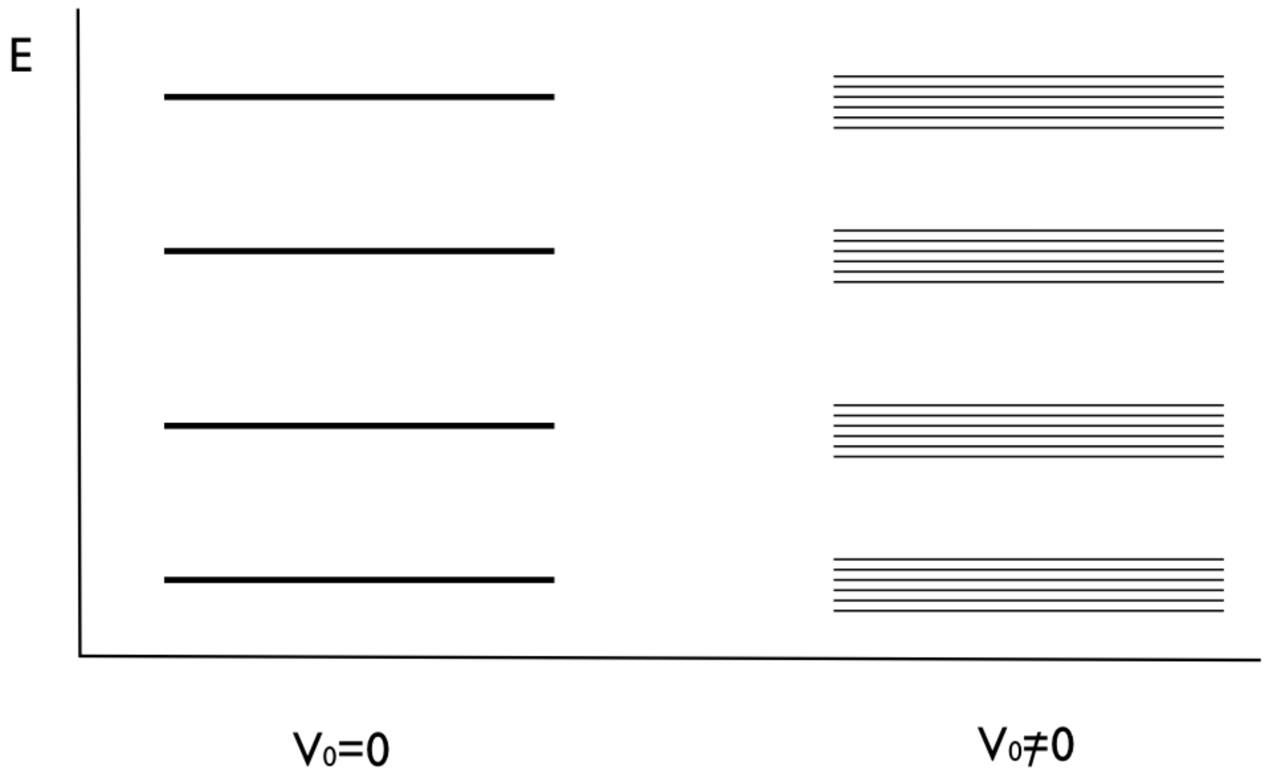
$$\sigma_{xy} = \frac{e^2}{2\pi} \nu$$

Quantization of Hall conductivity

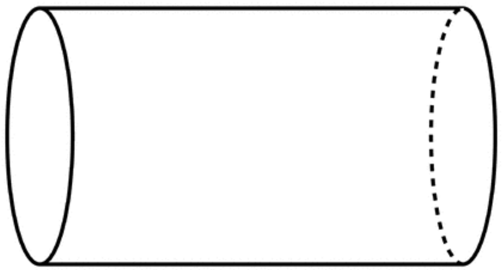
Band structure

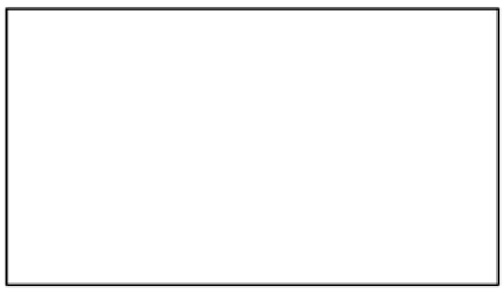
Periodic perturbation:

$$H'' = H' + V_0 \sin k\,\varphi_1 \sin k\,\varphi_2$$

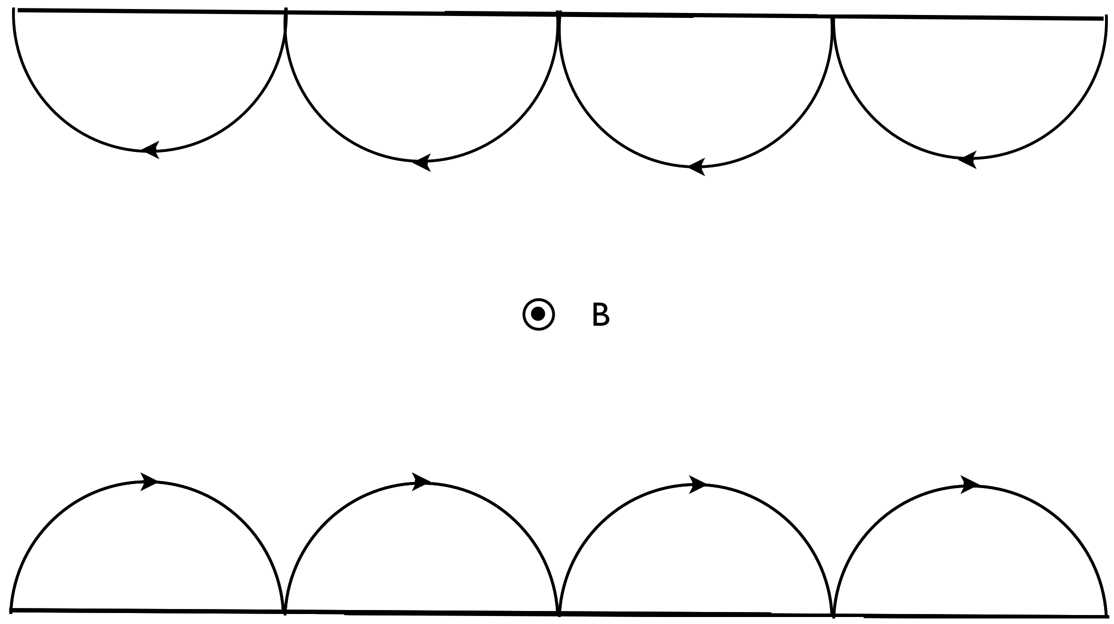


Hall effect with boundaries

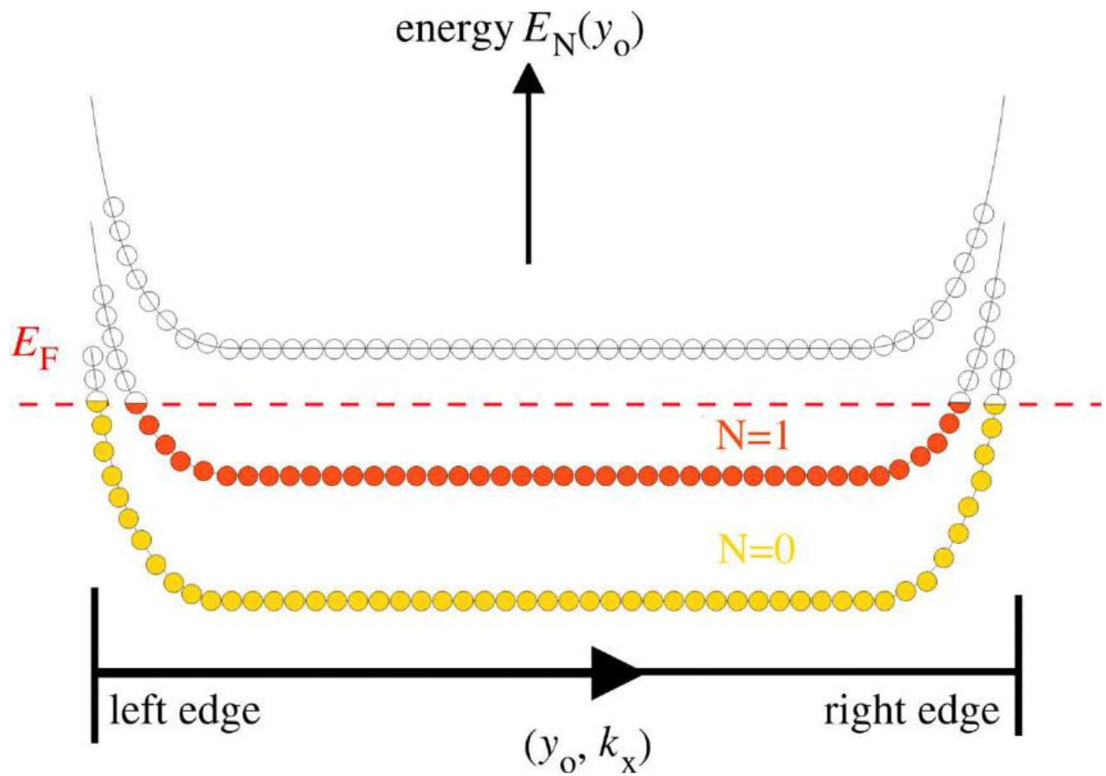




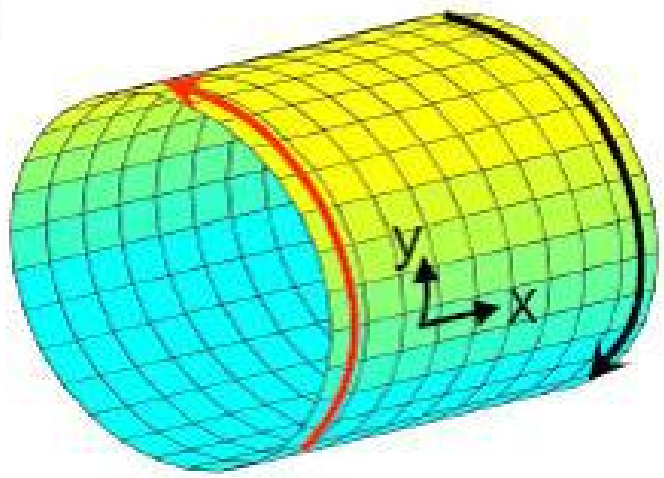
Hall effect with boundaries

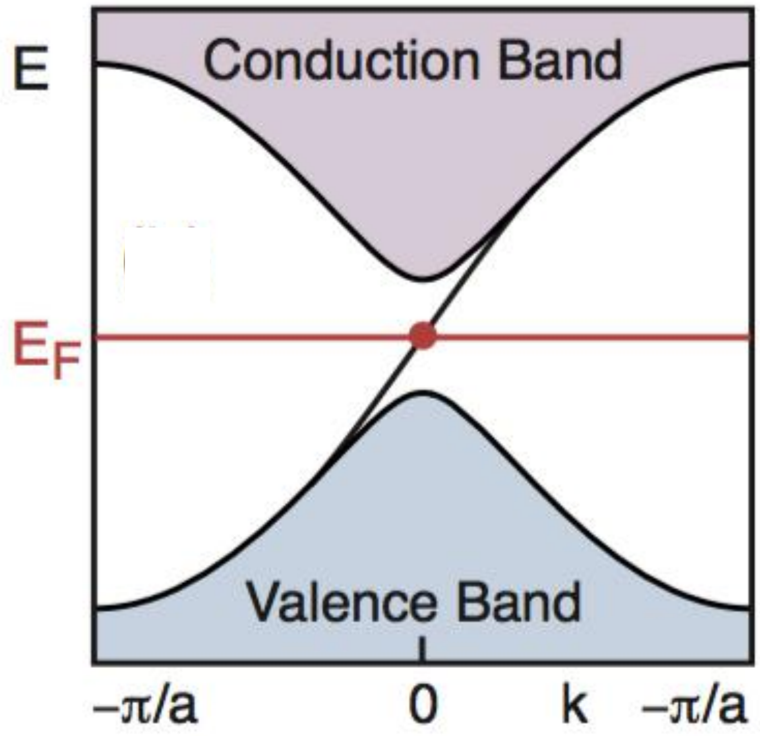


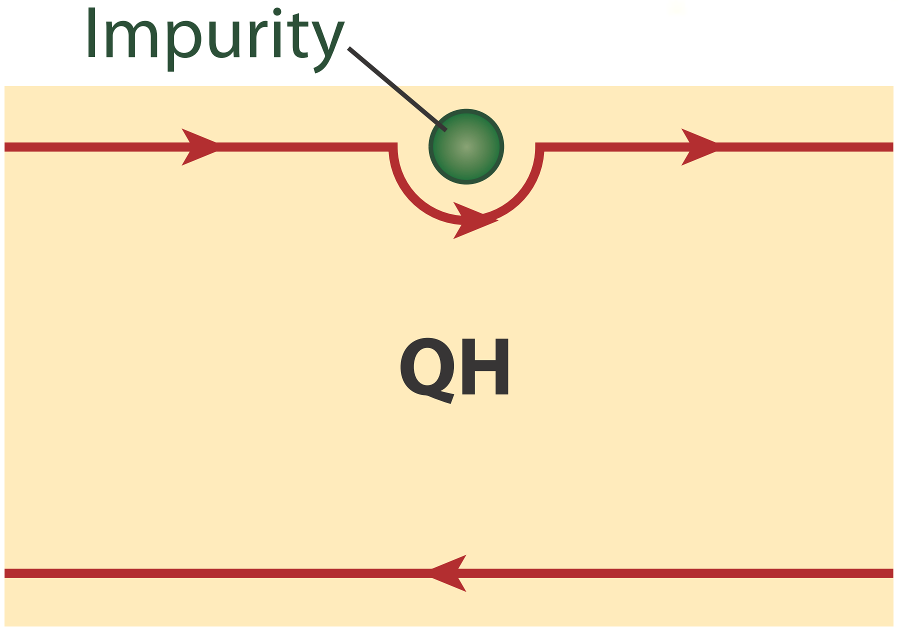
Finite size effects



Edge states and bands structure







Boundary Insulators

Boundary interactions $\Rightarrow$ Anderson localization

Edge states and Anomalous QH effect

Chern-class on the cylinder is not any more an integer but the quantization of the Hall conductivity can be associated to the number of edge states around the Fermi level which again is an integer quantum number which only depends on the number of ocuppied Landau levels

Edge states are chiral, due to the TR violation introduced by the magnetic field

Can edge states survive without magnetic field?

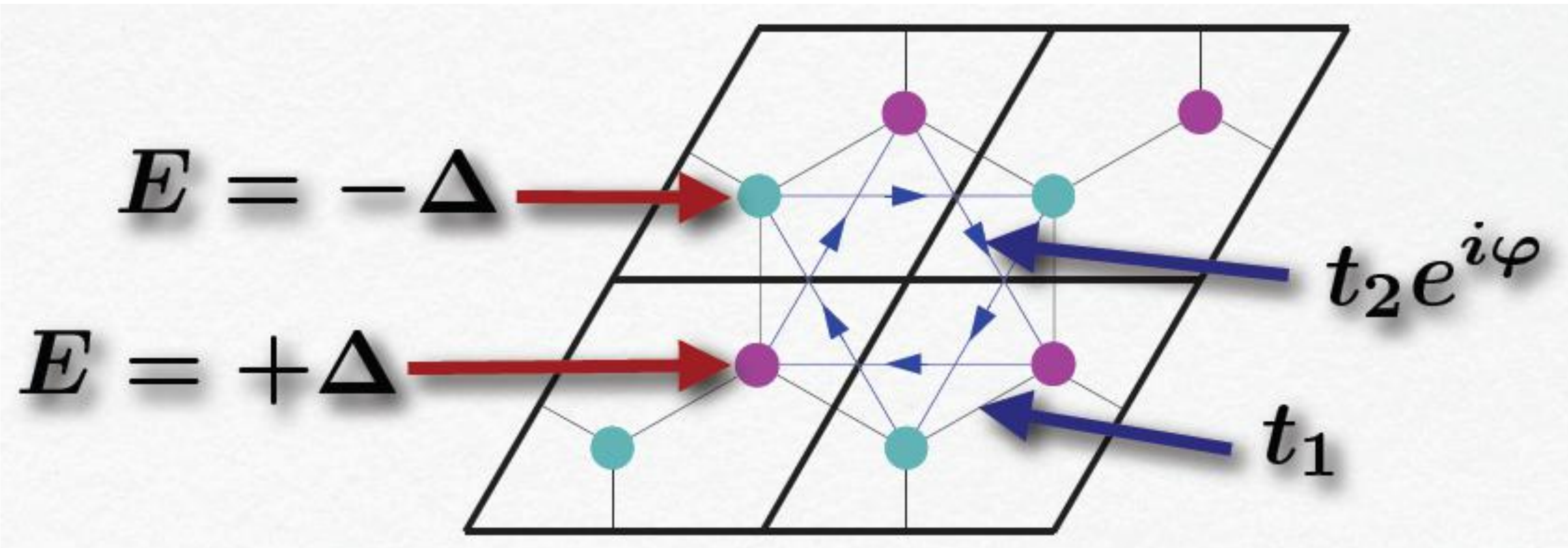
Haldane model and Anomalous QH effect

VOLUME 61, NUMBER 18 PHYSICAL REVIEW LETTERS 31 OCTOBER 1988

Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the "Parity Anomaly"

F. D. M. Haldane
Department of Physics, University of California, San Diego, La Jolla, California 92093
(Received 16 September 1987)

A two-dimensional condensed-matter lattice model is presented which exhibits a nonzero quantization of the Hall conductance σ^{xy} in the *absence* of an external magnetic field. Massless fermions *without spectral doubling* occur at critical values of the model parameters, and exhibit the so-called "parity anomaly" of (2+1)-dimensional field theories.

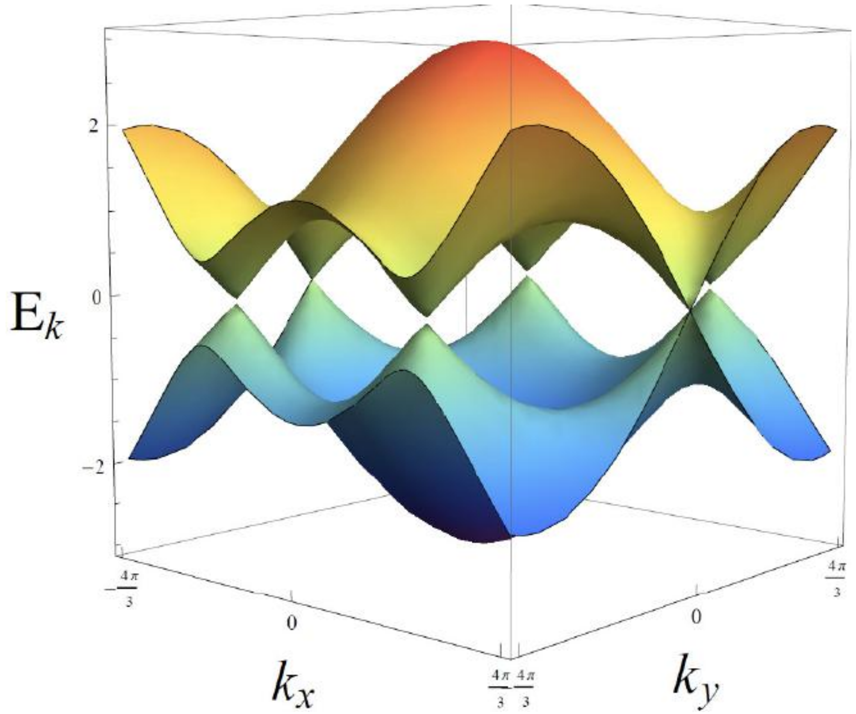


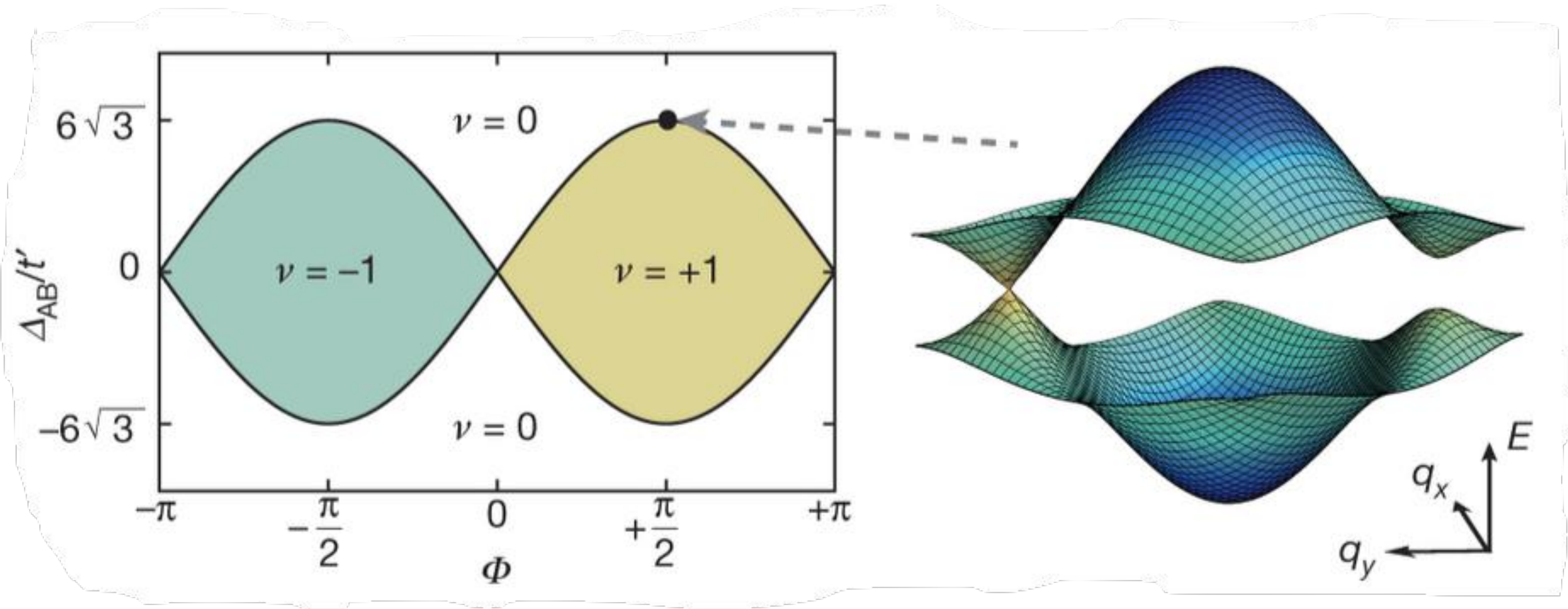
Haldane model and Anomalous QH effect

$$H = 2t_2 \cos\phi \sum_i \cos \mathbf{k}\cdot\mathbf{b}_i\, \mathbb{I} + t_1 \sum_i (\cos \mathbf{k}\cdot\mathbf{a}_i\ \sigma_1 + \sin \mathbf{k}\cdot\mathbf{a}_i\ \sigma_2)$$
$$+ (\Delta - 2t_2 \sin\phi \sum_i \sin \mathbf{k}\cdot\mathbf{b}_i)\ \sigma_3$$

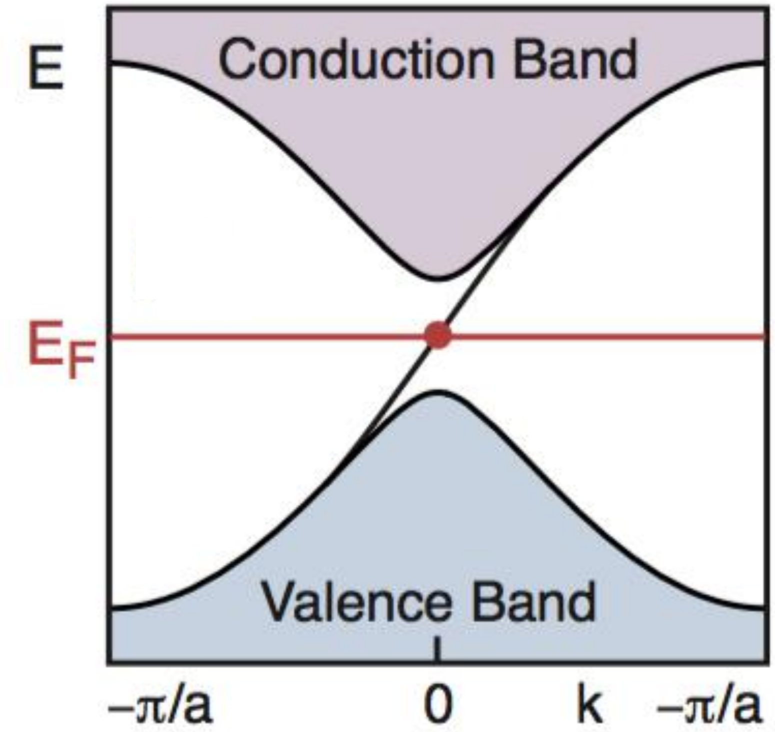
While the particular model presented here is unlikely to be directly physically realizable, it indicates that, at least in principle, the QHE can be placed in the wider

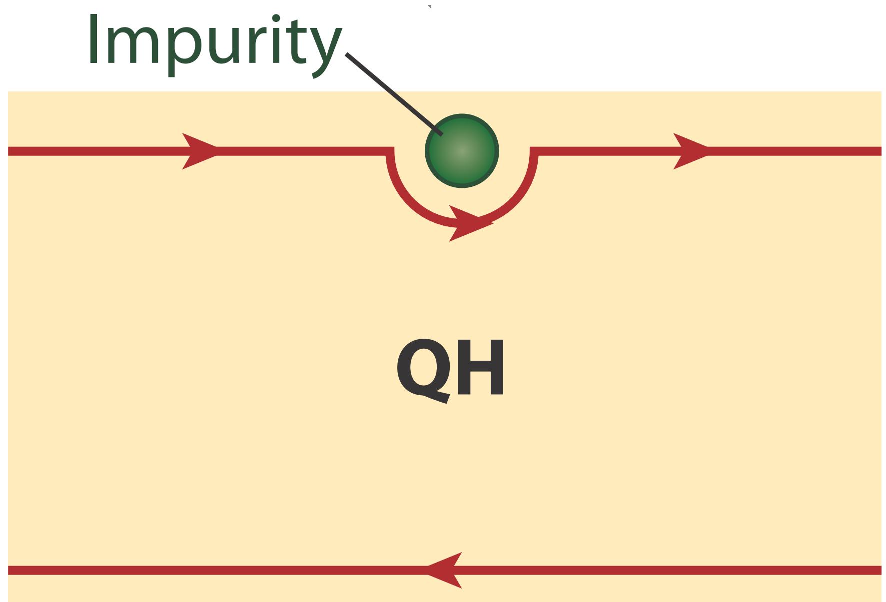
$$\Delta = t_2 \sin\phi = 0$$





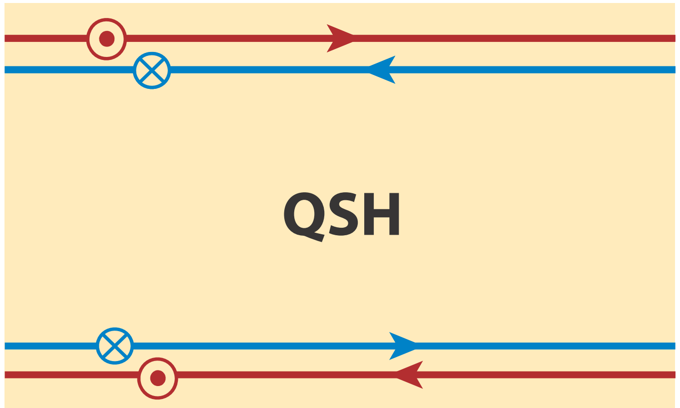
Edge states in Haldane model

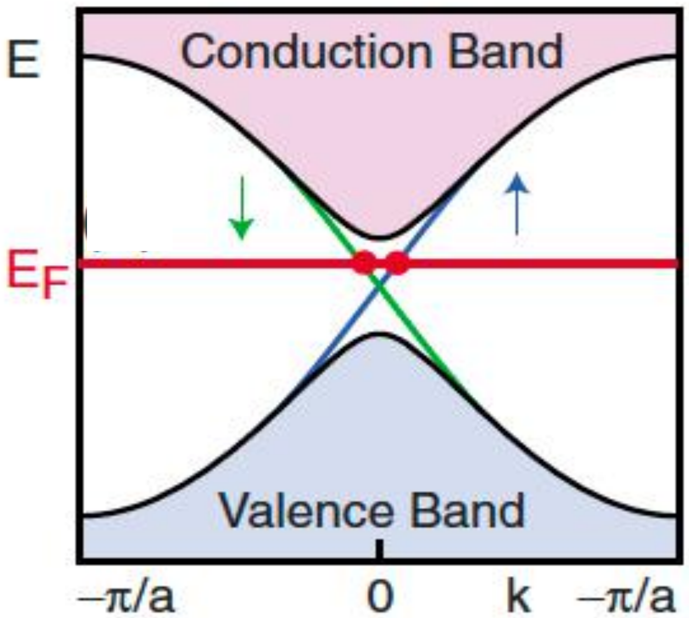




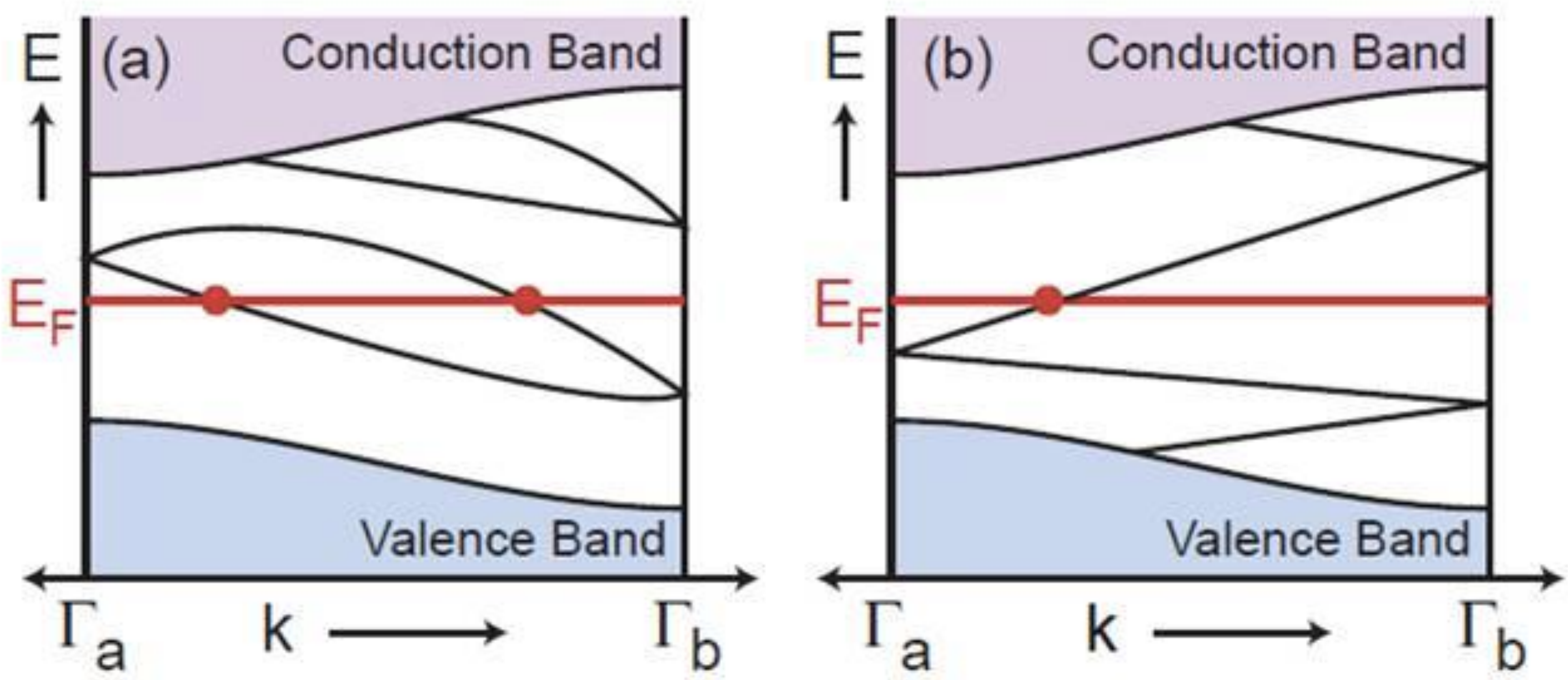
Quantum Spin Hall

Two copies of Haldane model $\phi = \pm\frac{\pi}{2}$ $S = \pm\frac{1}{2}$





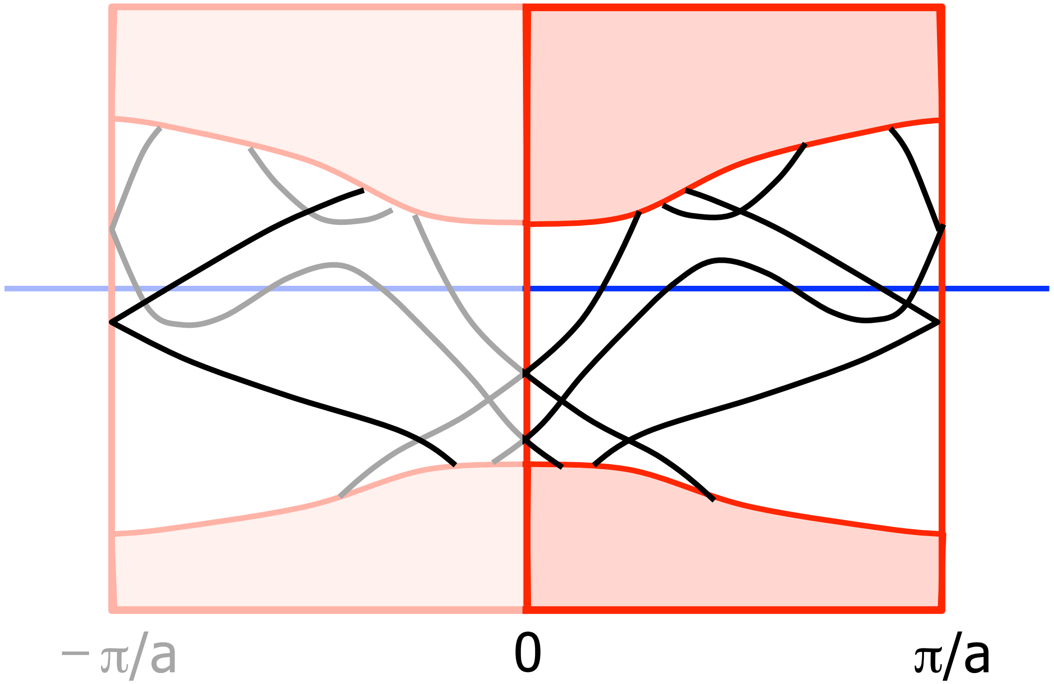
Topological Insulators



Normal Insulators

Topological Insulators

Kane-Mele Z_2 index



$Z_2 = N_{cross}$ (mod 2) = Invariant

$\mathbb{Z}_2$ Index

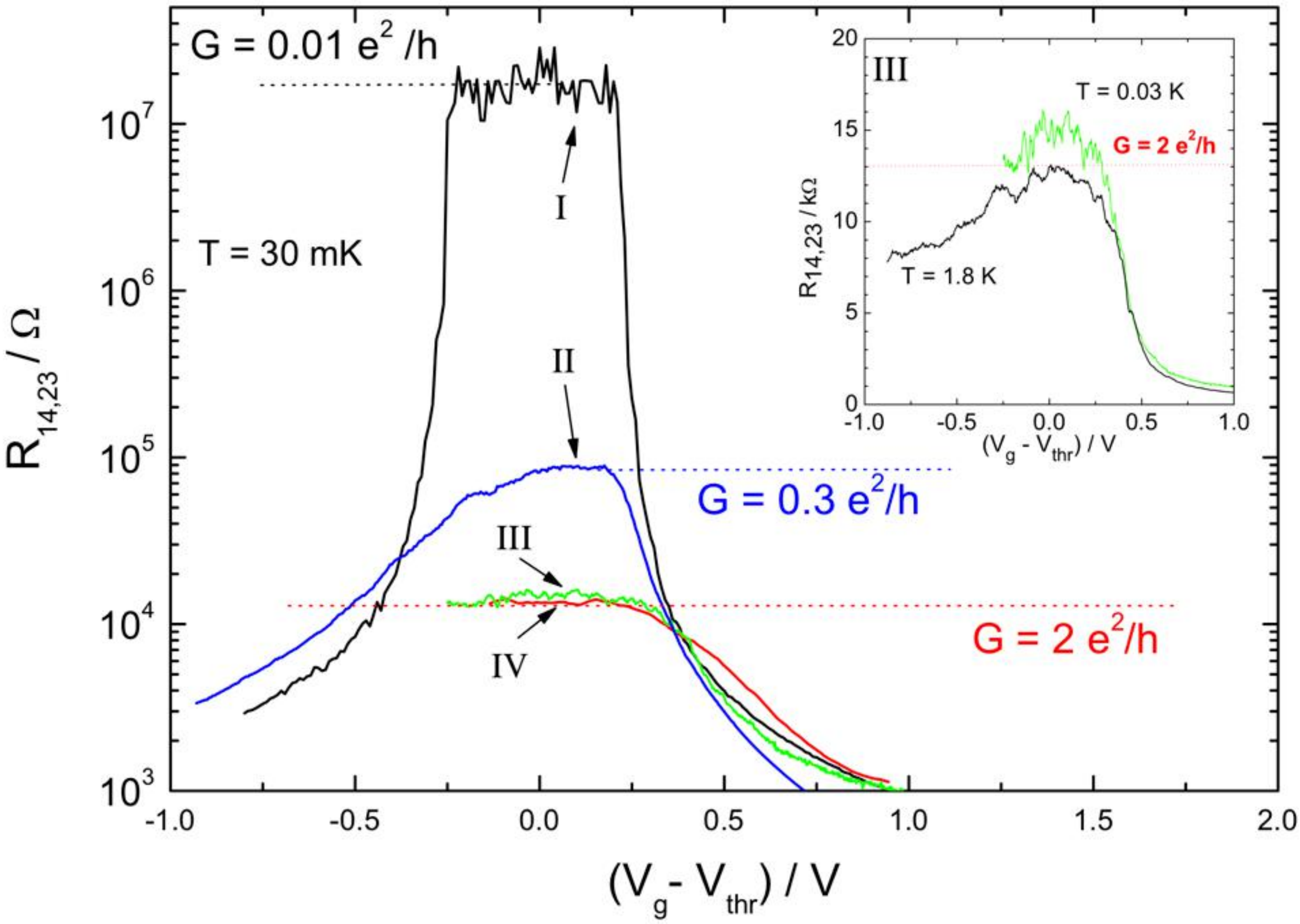
Time reversal matrix

$$w_{mn}(k) =< u_m(k)|\Theta|u_n(-k) > \qquad |u_n(k) > \text{ filled states}$$

$$w_{mn}(k) = -w_{nm}(-k)$$

For TR invariant k_a the matrix $w(k_a)$ is antisymmetric
$\mathbb{Z}_2$ invariant ν is defined by

$$(-1)^\nu = \prod_a \frac{Pf\,(w(k_a))}{\det w(k_a)} = \pm 1$$



G = 0.01 e^2 /h
T = 30 mK
I
II
III
IV
G = 0.3 e^2/h
G = 2 e^2/h
R_14,23 / Ω
(V_g - V_thr) / V
III
T = 0.03 K
G = 2 e^2/h
T = 1.8 K
R_14,23 / kΩ
(V_g - V_thr) / V

3D Topological Insulators

Four Z_2 indices

$$\nu_0 \ \nu_1 \ \nu_2 \ \nu_3$$

$$\nu_0 = \frac{1}{4\pi} \int \epsilon^{ijk} \left(\mathcal{A}_i^a \partial_j \mathcal{A}_k^a + \frac{2}{3} f_{abc} \mathcal{A}_i^a \mathcal{A}_j^b \mathcal{A}_k^c \right)$$

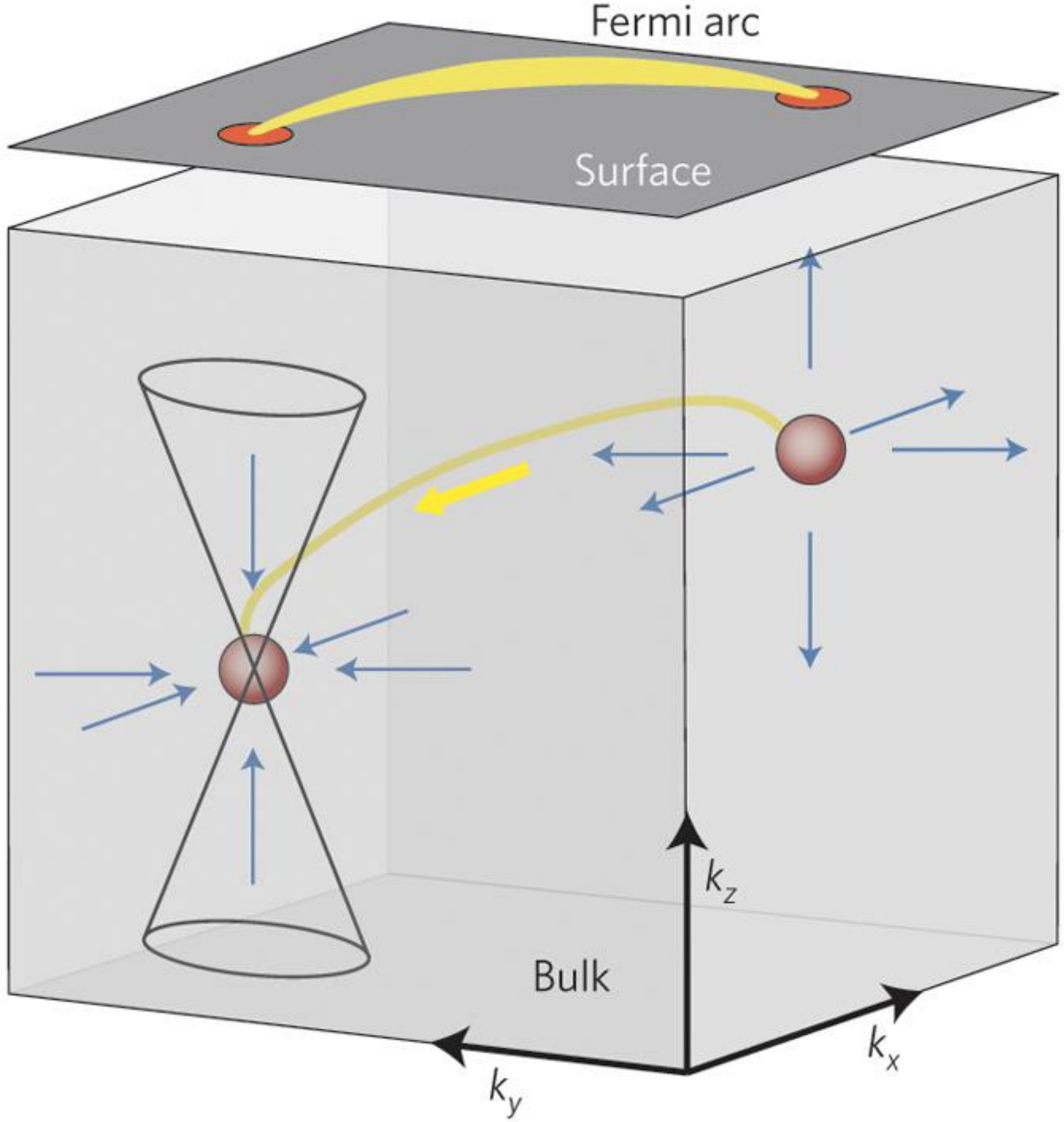
Weak topological insulators $\nu_0 = 2n$

Weak topological insulators $\nu_0 = 2n+1$

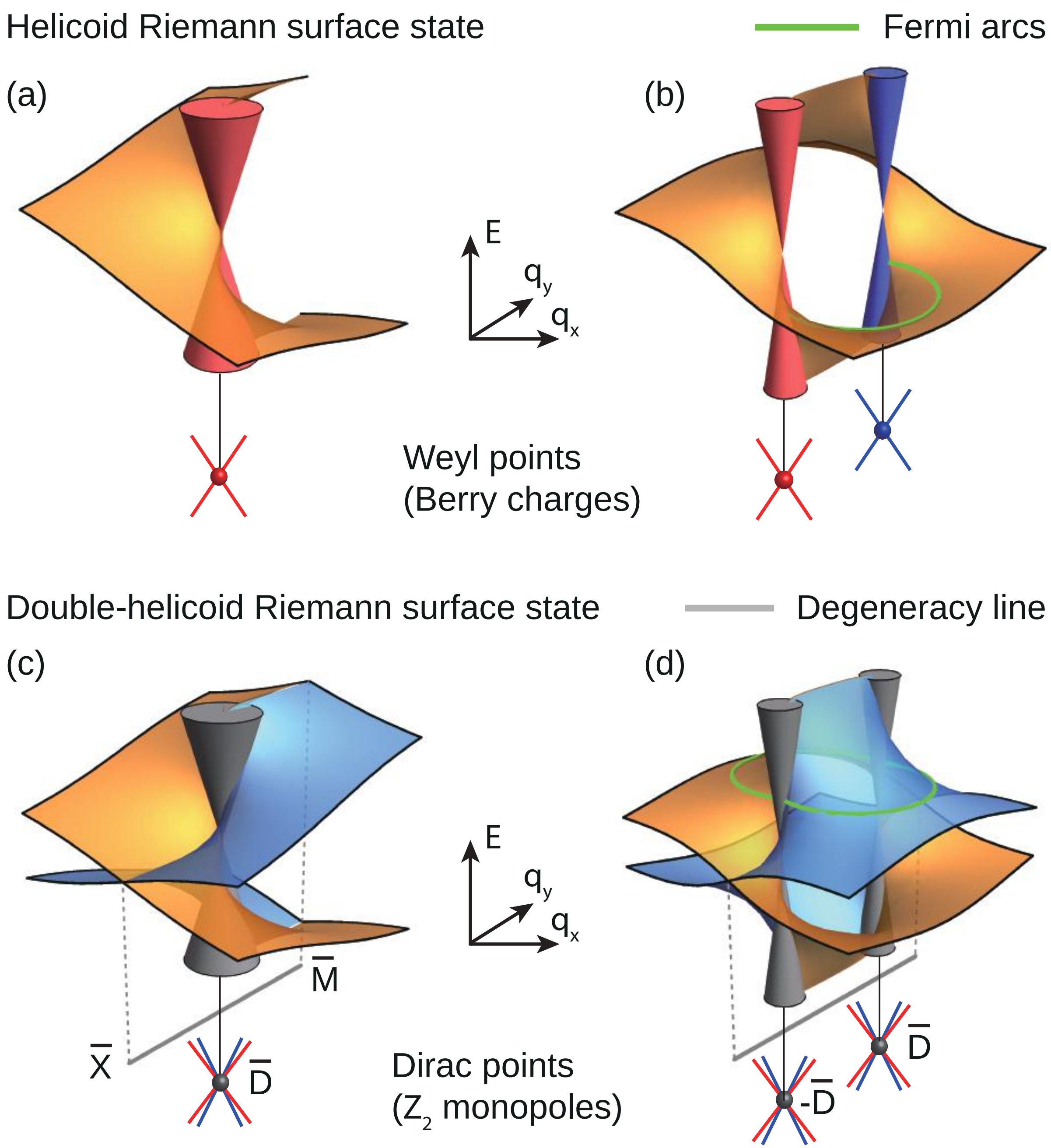
Non-trivial TR Bloch bundle

Topological Insulators

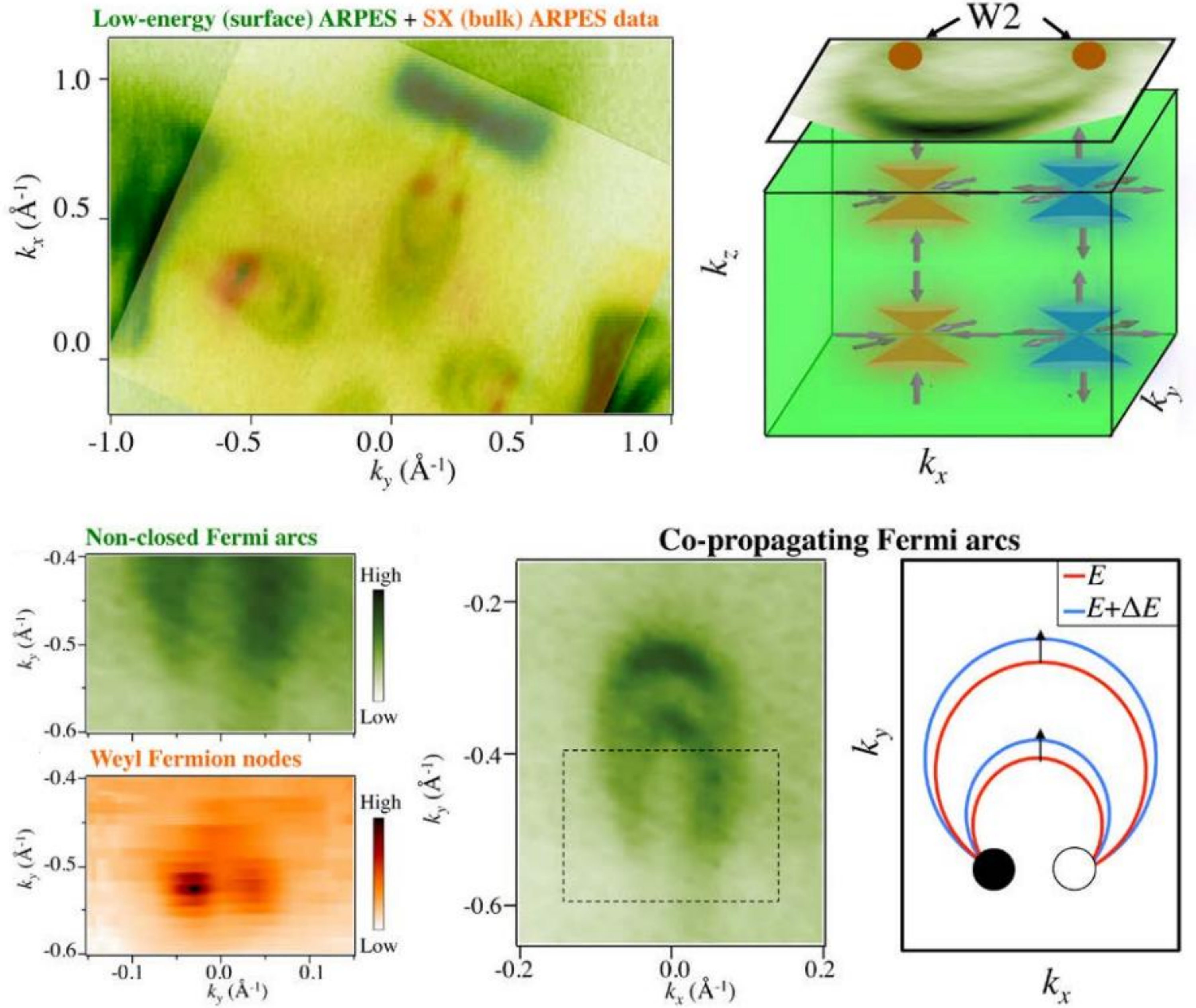
Weyl semimetals (2015)



Weyl semimetals (2016)



Weyl Fermion nodes and Topological Fermi arcs



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