LATTICE QCD:ACHIEVEMENTS AND CHALLENGES

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Outline

- Historical remarks
- Lattice formulation
- Some relevant results
  - Quark confinement
  - Spontaneous chiral symmetry breaking
  - Lattice phenomenology
  - Finite Temperature
  - Higgs models

Outline

- Challenges
  - Finite density QCD
  - θ-vacuum
- Conclusions and Outlook

Historical remarks

Standard Model

$SU(3) \times SU(2) \times U(1)$

Electroweak Sector

$SU(2) \times U(1)$

Strong Sector

$SU(3)$

Historical remarks

Standard Model

$SU(3) \times SU(2) \times U(1)$

Electroweak Sector

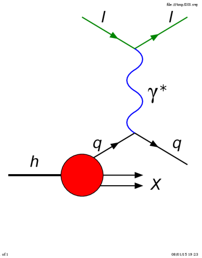
$SU(2) \times U(1)$

Strong Sector

$SU(3)$

Asymptotic Freedom!

- F. Wilczek, D. Gross, D. Politzer 1973 (2004 Nobel Prize)



Historical remarks

F. Wilczek

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- I'd like to thank Murray Gell-Mann and Gerard 'tHooft for not quite inventing everything, and so leaving us something to do. And finally I'd like to thank Mother Nature for her extraordinarily good taste, which gave us such a beautiful and powerful theory to discover.

Historical remarks

Non Perturbative Physics

- Hadron spectroscopy
- Spontaneous chiral symmetry breaking
- Quark confinement
- Chiral anomaly and Strong CP Problem
- Finite temperature and baryon density

K.G. Wilson Phys. Rev. D10 2445 (1974), Lattice Formulation
M. Creutz Phys. Rev. D21 2308 (1980), Continuum Physics

Hystorical remarks

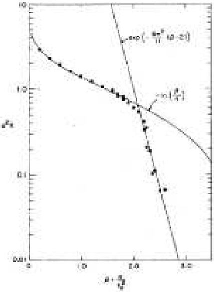

FIG. 6. The cutoff squared times the string tension as a function of β. The solid lines are the strong- and weak-coupling limits.

low $\beta = 2.1$ only loops of side 1 and 2 are significantly different from zero so we must include the loop of side 0 in the fit. Below $\beta = 1.6$ only the loop of side 1 is significant and we assume the area term C dominates. From Eq. (3.21) we iden-

behavior of Eq. (3.22) occurs rather sharply over a range of about 10% in β about $\beta = 2$. This appearance of the confinement mechanism occurs at

$$\frac{e_0^2}{4\pi} = 0.16\,. \qquad (5.1)$$

The rapid evolution out of the perturbative regime may be responsible for the remarkable phenomenological successes of the bag model.[21] High-temperature-series results,[18] as well as semiclassical treatments,[5] have also suggested an abrupt onset of confinement.

Our analysis allows a determination of the renormalization scale of the coupling in terms of the string tension. Using the observed asymptotic normalization

$$a^2K \underset{\beta\to\infty}{\sim} \exp\left(-\frac{6\pi^2}{11}(\beta - 2)\right), \qquad (5.2)$$

we can solve for e_0^2 to give

$$\frac{e_0^2}{4\pi} \underset{a\to 0}{\sim} \frac{3\pi}{11\ln(1/a\Lambda)}, \qquad (5.3)$$

where the renormalization scale is

$$\Lambda \approx \sqrt{K}\exp\left(-\frac{6\pi^2}{11}\right) = \frac{1}{200}\sqrt{K}\,. \qquad (5.4)$$

Thus we see the appearance of a rather large di-

Historical remarks

M. Creutz Phys. Rev. Lett. 43 553 (1979), Quark Confinement

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spin systems are confirmed.[4]

In this note I report results on Monte Carlo studies of SU(2) gauge theory. To show the critical nature of four dimensions I ran these simulations for both four- and five-dimensional lattices. I also make comparisons with the Abelian group SO(2) [isomorphic to U(1)]. I work with pure gauge fields on the assumption that the addition of a few fermion species represents a perturbation that will not spoil confinement. Although the group of physical interest is SU(3), I study SU(2) because of its simpler structure. As confinement is connected with disorder in the lattice formulation, and as adding more degrees of freedom should increase disorder, confinement with SU(2) gauge fields should imply confinement with SU(3).

The system is formulated on a hypercubical lattice. Associated with the link joining any pair of nearest-neighbor sites i and j is an element U_{ij} of the gauge group (i and j label sites and should not be confused with the implicit matrix indices on the group elements). The wave function of a particle traversing the respective link undergoes an internal-symmetry rotation corresponding to U_{ij}. The reverse path gives the conjugate rotation

$$U_{ji} = (U_{ij})^{-1}, \quad (1)$$

where the inverse is in the group sense. The quantum theory is defined via the path integral

$$Z = \int \Big(\prod_{(i,j)} dU_{ij} \Big) e^{-\beta S(U)}, \quad (2)$$

where the integral includes all links and uses the invariant group measure. The action is that defined by Wilson,

$$S(U) = \sum_{\square} S_{\square}, \quad (3)$$

where the sum extends over all elementary squares or "plaquettes" □ and

$$S_{\square} = 1 - \tfrac{1}{2}\operatorname{Tr}(U_{ij}U_{jk}U_{kl}U_{li}). \quad (4)$$

Here i, j, k, and l are some labeling of the sites going around the square □. The normalization is such that for the groups SU(2) and SO(2) any plaquette contributes a number between zero and two to the action. As shown by Wilson,[1] this action reduces in the classical continuum limit to the usual gauge-theory action with β proportional to

The Monte Carlo algorithm consists of successively touching a heat bath to each link of the lattice while holding fixed the group elements on the remaining links. Repeating this procedure will eventually produce a sequence of states which simulates an ensemble of such systems in thermal equilibrium.[5] Green's functions for the quantum theory follow from correlation functions in the states of the ensemble.

Beginning in some initial configuration, we pass through the entire lattice varying one link at a time. At each link's turn, a new group element g is selected to occupy that position. This choice is made randomly from the entire gauge group with weighting proportional to the Boltz-

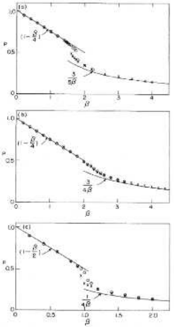


FIG. 1. The average plaquette as a function of β as

Lattice Formulation

- Feynman Path Integral Quantization (1948)
- Wick Rotation to Euclidean Space
- Space-time Discretization

Quantum Mechanics: one-dimensional harmonic oscillator

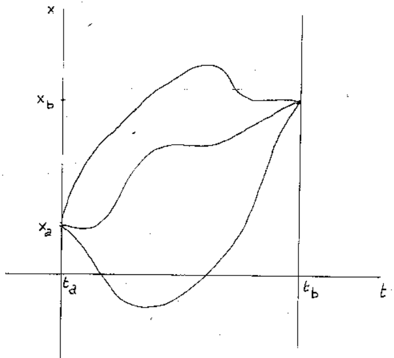
$$\mathcal{L}_m = \frac{1}{2}\left(\frac{dx}{dt}\right)^2 - \frac{1}{2}\omega^2 x^2$$

$$\mathcal{S}_m = \int \mathcal{L}_m\left(x\left(t\right)\right) dt$$

$$\psi\left(x_b, t_b\right) = \int dx_a \langle x_b | e^{-\frac{i}{\hbar}H(t_b - t_a)} | x_a \rangle \psi\left(x_a, t_a\right)$$

Lattice Formulation

$$\mathcal{Z} \quad = \quad \langle x_b | e^{-\frac{i}{\hbar}H(t_b - t_a)} | x_a \rangle = \sum_{paths} e^{\frac{i}{\hbar}\mathcal{S}_m}$$



Lattice Formulation

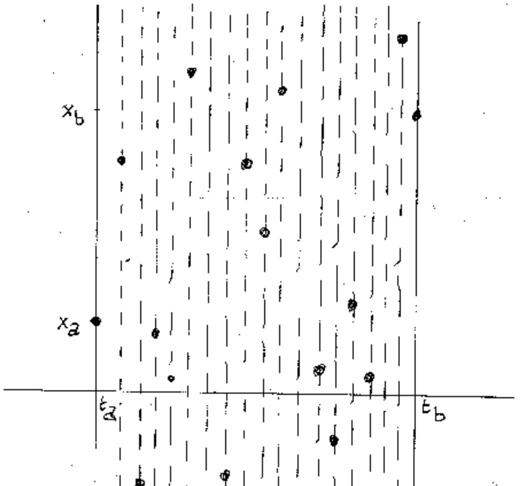
Euclidean time $t = -i\tau$

$$\mathcal{Z} = \langle x_b | e^{-\frac{1}{\hbar} H(\tau_b - \tau_a)} | x_a \rangle = \sum_{paths} e^{-\frac{1}{\hbar}\mathcal{S}}$$

$$\mathcal{L} = \frac{1}{2}\left(\frac{dx}{d\tau}\right)^2 + \frac{1}{2}\omega^2 x^2$$

$$\mathcal{S} = \int_{\tau_a}^{\tau_b} \mathcal{L}(\tau)\, d\tau$$

Lattice Formulation



Lattice Formulation

Discretization rules

$$\int d\tau \rightarrow \epsilon \sum_i$$

$$\frac{dx}{d\tau} \rightarrow \frac{x_{i+1} - x_i}{\epsilon}$$

$$\mathcal{S} = \frac{1}{2\epsilon} \sum_i (x_{i+1} - x_i)^2 + \frac{\epsilon}{2} \sum_i \omega^2 x_i^2$$

$$\mathcal{Z} = \int_{-\infty}^{+\infty} \prod_i dx_i e^{-\frac{1}{\hbar}\mathcal{S}[x_1, x_2, \ldots\ldots x_N]}$$

Lattice Formulation

$$\mathcal{Z} \;=\; \langle x_b | e^{-\frac{1}{\hbar} H(\tau_b - \tau_a)} | x_a \rangle$$

Periodic Boundary Conditions Partition Function

$$\mathcal{Z} \;=\; tr\left[e^{-\frac{\epsilon}{\hbar} N \mathcal{H}(\epsilon)} \right]$$

$$\mathcal{H}(\epsilon) \;=\; \mathcal{H} + \mathcal{O}(\epsilon) = \frac{1}{2}\left(\mathbf{p}^2 + \omega^2 \mathbf{x}^2\right) + \mathcal{O}(\epsilon)$$

$$\mathcal{Z} \;=\; \sum_{j=0}^{\infty} e^{-\frac{\epsilon}{\hbar} N E_j} = e^{-\frac{\epsilon}{\hbar} N E_0} \left(1 + \sum_{j=1}^{\infty} e^{-\frac{\epsilon}{\hbar} N \left(E_j - E_0\right)} \right)$$

Lattice Formulation

$$\langle x_i x_{i+r} \rangle_c \quad = \quad \sum_{k=1}^{\infty} \langle 0|\mathbf{x}|k\rangle \langle k|\mathbf{x}|0\rangle e^{-\frac{\epsilon}{\hbar} r(E_k - E_0)}$$

$$\xi \quad = \quad \frac{\hbar}{\epsilon\,(E_1 - E_0)} \to \infty$$

Critical Point!

Lattice Formulation

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Critical Point!

- Statistical Mechanics $\iff$ Quantum Theory
- Hamiltonian $\iff$ Euclidean Action
- T $\iff$ $\hbar$
- Free Energy Density $\iff$ Vacuum Energy density
- Correlation Length ξ $\iff$ Inverse Mass Gap
- Critical point ($\xi \to \infty$) $\iff$ Quantum Continuum Limit $\epsilon \to 0$

Lattice Formulation

Quantum Field Theory: space-time lattice

- Scalar Field
- Gauge Field
- Fermion Field

Scalar Field

$$\mathcal{L}_m = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 - \lambda\phi^4$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial_\mu\phi + \frac{m^2}{2}\phi^2 + \lambda\phi^4$$

Derivatives $\rightarrow$ Finite differences
Integral $\rightarrow$ Sum

Lattice Formulation

$$\mathcal{S} = \int \mathcal{L} d^4x$$

$$\mathcal{S} = \frac{a^2}{2} \sum_{n,\mu} (\phi(n+\mu) - \phi(n))^2 + \frac{m^2 a^4}{2} \sum_n \phi(n)^2 + \lambda a^4 \sum_n \phi(n)^4$$

Lattice Formulation

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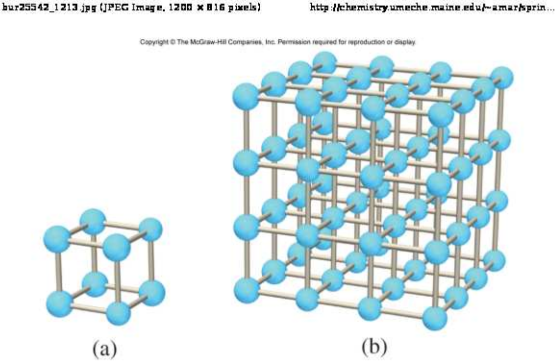


Lattice Formulation

$$\mathcal{Z} = \int \prod_n d\phi(n) e^{-\frac{1}{\hbar}\left[\frac{a^2}{2}\sum_{n,\mu}(\phi(n+\mu)-\phi(n))^2+\frac{m^2 a^4}{2}\sum_n \phi(n)^2+\lambda a^4 \sum_n \phi(n)^4\right]}$$

Lattice Formulation

Gauge Field

Formulation based on the use of the gauge group variables

$$U_{path}(A, B) = e^{ig \int_{path} \lambda_b A_\mu^b dx_\mu}$$

Lattice Formulation

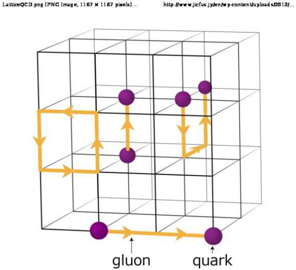
Gauge Field

Formulation based on the use of the gauge group variables

$$U_{path}(A,B) = e^{ig\int_{path}\lambda_b A_\mu^b dx_\mu}$$

Elementary string

$$U_\mu(n) = e^{i\frac{1}{2}ga\lambda_b A_\mu^b(n)}$$



Lattice Formulation

Lattice pure gauge action

$$\mathcal{S}_G(U) = -\beta \sum_{n,\mu,\nu} tr\left[U_\mu(n)\, U_\nu(n+\mu)\, U_\mu^+(n+\nu)\, U_\nu^+(n)\right] + c.c.$$

$$\beta \sim \frac{1}{g^2}$$

Fermion Field

$$\mathcal{S}_F\left(\psi, \bar{\psi}, A\right) = \sum_{f=1}^{N_f} \int dx^4 \bar{\psi}^f(x) \left(\gamma_\mu\left(\partial_\mu + iA_\mu(x)\right) + m^f\right) \psi^f(x)$$

$$A_\mu(x) = \frac{1}{2} \sum_{b=1}^{8} A_\mu^b(x)\, \lambda_b$$

Lattice Formulation

$$\mathcal{S}_F\left(\psi,\bar{\psi},U\right) = \sum_{f=1}^{N_f} a^4 \sum_{n,m} \bar{\psi}^f(n)\,\Delta_{nm}\psi^f(m)$$

$$\begin{aligned}\bar{\psi}\Delta\psi &= \left(m+\frac{4}{a}\right)\sum_n \bar{\psi}(n)\,\psi(n)\\ &-\left(m+\frac{4}{a}\right)\kappa\sum_{n,\mu}\bar{\psi}(n)\left(1-\gamma_\mu\right)U_\mu(n)\,\psi(n+\mu)\\ &-\left(m+\frac{4}{a}\right)\kappa\sum_{n,\mu}\bar{\psi}(n+\mu)\left(1+\gamma_\mu\right)U_\mu^{+}(n)\,\psi(n)\end{aligned}$$

$$\kappa = \frac{1}{8+2ma}$$

Lattice Formulation

$$\mathcal{Z} = \int \prod_{n,\mu} d\bar{\psi}(n)\, d\psi(n)\, dU_\mu(n) e^{-\mathcal{S}_F\left(\psi,\bar{\psi},U\right)-\mathcal{S}_G(U)}$$

$$\mathcal{Z} = \int \prod_{n,\mu} dU_\mu(n) \det \Delta(U)\, e^{-\mathcal{S}_G(U)}$$

Quark propagators in a background gauge field

$$\langle \bar{\psi}(m)\, \psi(n) \rangle_U = \Delta^{-1}_{nm}$$

Lattice Formulation

- Partition Function $\Longleftrightarrow$ Path Integral
- Mean Value $\Longleftrightarrow$ Vacuum Expectation Value
- Low Temperature Expansion $\Longleftrightarrow$ Weak Coupling Expansion
- High Temperature Expansion $\Longleftrightarrow$ Strong Coupling Expansion

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- Numerical Monte Carlo Simulations!

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- High Temperature Expansion $\Longleftrightarrow$ Strong Coupling Expansion
- Numerical Monte Carlo Simulations!
- Quantum Continuum Limit $\rightarrow$ Renormalization

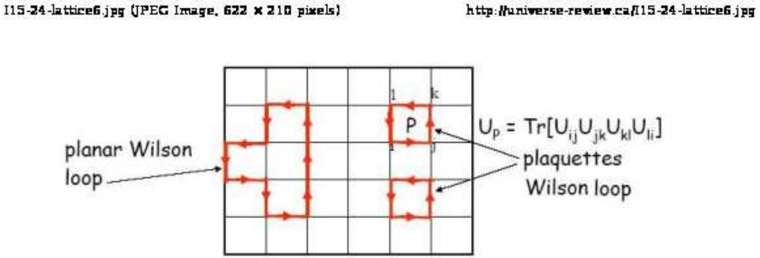
Some relevant results

Quark confinement

K.G. Wilson Phys. Rev. D10 2445 (1974)

Wilson loop (Wilson-Wegner loop)

$$\mathcal{W}_C(U) = tr\left[\prod_{(n,\mu)\epsilon C} U_\mu(n)\right]$$



Some relevant results

Quark confinement

Static quarks

$$\langle \mathcal{W}_C \rangle = \frac{\int \prod_{m,\nu} dU_\nu(m) \mathcal{W}_C(U)\, e^{-\mathcal{S}_G(U)}}{\int \prod_{m,\nu} dU_\nu(m) e^{-\mathcal{S}_G(U)}}$$

Fixing the temporal gauge and $T \to \infty$

$$\langle \mathcal{W}_{R,T} \rangle \sim e^{-V(R)T}$$

$V(R)$ Energy of a quark-antiquark pair at distance R
Area low! for large Wilson loops $V(R) = cR$

M. Creutz Phys. Rev. Lett. 43 553 (1979)

Some relevant results

Spontaneous chiral symmetry breaking

SCSB plays a fundamental role in QCD:

- Pions have unexpectedly small masses
- We do not see degenerate masses for parity partners in the baryon sector

One flavor case $U(1)_A$

$$\psi = e^{i\alpha\gamma_5}\psi' \qquad \bar{\psi} = \bar{\psi}' e^{i\alpha\gamma_5}$$

$$\bar{\psi}\psi \quad = \quad \bar{\psi}' e^{i2\alpha\gamma_5}\psi'$$

Several flavors $SU(N)_A \times U(1)_A$

$$\psi = e^{i\bar{\alpha}\gamma_5\bar{T}}\psi' \qquad \bar{\psi} = \bar{\psi}' e^{i\bar{\alpha}\gamma_5\bar{T}}$$

$$\psi = e^{i\alpha\gamma_5}\psi' \qquad \bar{\psi} = \bar{\psi}' e^{i\alpha\gamma_5}$$

Some relevant results

Spontaneous chiral symmetry breaking

Order Parameter $\langle\bar{\psi}\psi\rangle$
STRONG analytical (strong coupling and $1/d$ expansions) and numerical evidence for SCSB

Difficulties:

- Wilson Fermions break explicitely CS
- Staggered Kogut-Susskind fermions, Ginsparg-Wilson fermions
- Grassmann variables can not be simulated in a computer
  - **V. Azcoiti, A. Cruz, E. Dagotto, A. Moreo, A. Lugo** Phys. Lett. B175 202 (1986)
    SCSB in strongly coupled compact QED
  - **T. Banks, A. Casher** Nucl. Phys. B169 103 (1980)
    $\langle\bar{\psi}\psi\rangle = \pi\rho(0)$
  - **V. Azcoiti, V. Laliena, X.Q. Luo** Phys. Lett. B354 111 (1995)
    Computation of the $p.d.f.$ of $\langle\bar{\psi}\psi\rangle$

Some relevant results

Lattice phenomenology

- Quenched approximation neglects the contribution of internal quark loops

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- Full two-color QCD with dynamical fermions, $\rho - \omega$ mass splitting
  - **V. Azcoiti, A. Nakamura** Phys. Rev. D27 (RC) 2559 (1983)
- Full three-color QCD with dynamical fermions
  - **H.W. Hamber** Nucl. Phys. B251 [FS13] 182 (1985)

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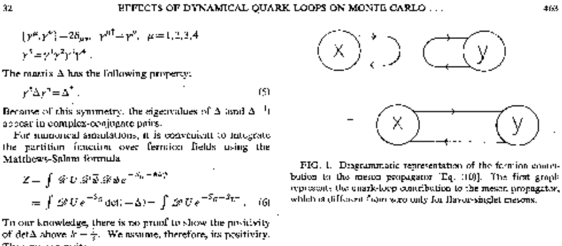
32 EFFECTS OF DYNAMICAL QUARK LOOPS ON MONTE CARLO ... 463

$$\{\gamma^\mu,\gamma^\nu\}=2\delta_{\mu\nu},\quad \gamma^{\mu\dagger}=\gamma^\mu,\quad \mu=1,2,3,4$$

$$\gamma^5=\gamma^1\gamma^2\gamma^3\gamma^4 .$$

The matrix Δ has the following property:

$$\gamma^5\Delta\gamma^5=\Delta^\dagger . \quad (5)$$

Because of this symmetry, the eigenvalues of Δ (and Δ^{-1}) appear in complex-conjugate pairs.

For numerical simulations, it is convenient to integrate the partition function over fermion fields using the Matthews-Salam formula

$$Z=\int \mathscr{D}U\,\mathscr{D}\bar\psi\,\mathscr{D}\psi\, e^{-S_G-\bar\psi\Delta\psi}$$

$$=\int \mathscr{D}U\, e^{-S_G}\det(-\Delta)=\int \mathscr{D}U\, e^{-S_G-S_F} , \quad (6)$$

To our knowledge, there is no proof to show the positivity of $\det\Delta$ above $k=\frac{1}{5}$. We assume, therefore, its positivity.

FIG. 1. Diagrammatic representation of the fermion contribution to the meson propagator [Eq. (10)]. The first graph represents the quark-loop contribution to the meson propagator, which is different from zero only for flavor-singlet mesons.

Some relevant results

Lattice phenomenology

S. Durr et al., Science 322 1224 (2008)

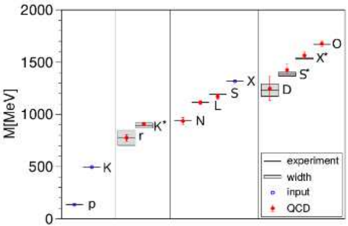

Figure 3: The light hadron spectrum of QCD. Horizontal lines and bands are the experimental values with their decay widths. Our results are shown by solid circles. Vertical error bars represent our combined statistical (SEM) and systematic error estimates. π, K and Ξ have no error bars, because they are used to set the light quark mass, the strange quark mass and the overall scale, respectively.

Some relevant results

Finite Temperature

Partition function of a quantum system in a heat bath with temperature T

$$\mathcal{Z}(T) = tr\left[e^{-\frac{1}{T}\mathbf{H}}\right]$$

Transfer Matrix at Euclidean time $\tau = \epsilon N$

$$\mathcal{Z}(\tau) = tr\left[e^{-aN\mathbf{H}}\right]$$

$$\mathcal{Z}(T) = \mathcal{Z}(\tau)_{\tau=\frac{1}{T}}$$

Some relevant results

Finite Temperature

Full QCD with dynamical fermions Polyakov loop is not an order parameter for the deconfining phase transition

Some relevant results

Finite Temperature

Full QCD with dynamical fermions Polyakov loop is not an order parameter for the deconfining phase transition

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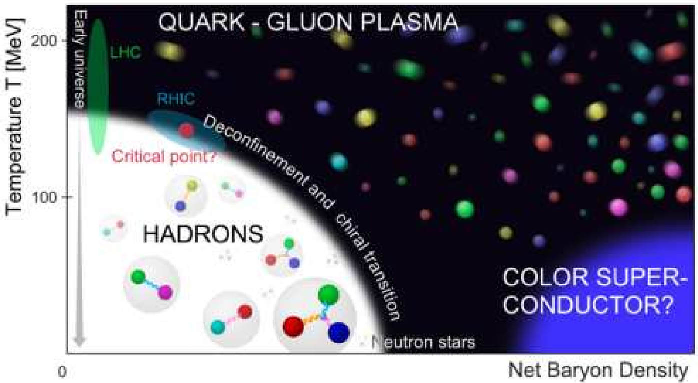

Some relevant results

Higgs models

One more field were the non-perturbative approach of lattice field theory has been successfully applied is the study of gauge-Higgs models. The analysis of the phase diagram, critical points and of the stability of classical topologically non trivial configurations under quantum fluctuations are some examples

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- Magnetic Monopole Excitations in the Georgi-Glashow Model
  - **V. Azcoiti, A. Cruz, G. Di Carlo, A.F. Grillo, A. Tarancon** Europhys. Lett. 9 (1) 23 (1989)

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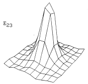

Fig. 2-a

Some relevant results

Higgs models

- Critical behavior of the U(1)-Higgs model in the confining-Higgs region
  - **J.L. Alonso, V. Azcoiti, I. Campos, J.C. Ciria, A. Cruz, D. Iñiguez, F. Lesmes, C. Piedrafita, A. Rivero, A. Tarancon, D. Badoni, L.A. Fernandez, A. Muñoz-Sudupe, J.J. Ruiz-Lorenzo, A. Gonzalez-Arroyo, P. Martinez, J. Pech, P. Tellez** Nucl. Phys. B405 574 (1993)

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  - **J.L. Alonso, V. Azcoiti, I. Campos, J.C. Ciria, A. Cruz, D. Iñiguez, F. Lesmes, C. Piedrafita, A. Rivero, A. Tarancon, D. Badoni, L.A. Fernandez, A. Muñoz-Sudupe, J.J. Ruiz-Lorenzo, A. Gonzalez-Arroyo, P. Martinez, J. Pech, P. Tellez** Nucl. Phys. B405 574 (1993)
- BSM physics: composite Higgs as an alternate explanation to the elementary scalar Higgs in the SM description. Strongly coupled gauge theories
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Challenges

Feynman path integral quantization + Euclidean formulation

$$\mathcal{Z} = \int [d\phi]\, e^{-\mathcal{S}(\phi, m, g, a)}$$

$\mathcal{S}(\phi, m, g, a)$ Real number!

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QCD

$$\mathcal{Z} \;=\; \int \left[d\bar{\psi}\right] [d\psi]\, [d\,U]\, e^{-\bar{\psi}\Delta(U)\psi - S_{PG}(U)}$$

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Challenges

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UNFORTUNATELY NO

- Finite Density QCD
- Topological Actions: θ-Vacuum, the Strong CP Problem and axion physics

Challenges

Finite Density QCD

LHC (CERN) and RHIC (BNL) Heavy Ion Collision Experiments:
test for new matter phases

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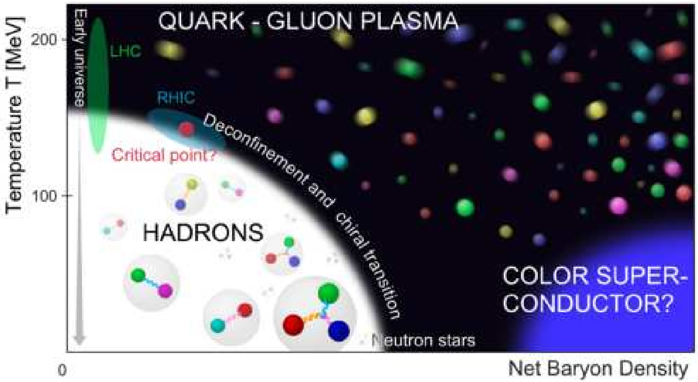

Challenges

Finite Density QCD

Standard way of introducing a chemical potential μ coupled to the baryon number operator:

$$U_4(n) \qquad e^{\mu} U_4(n)$$

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The fermion determinant $\det \Delta(m, a, \mu, U)$ which appears in the path integral integration measure becomes complex!

Severe Sign Problem: standard numerical simulation methods based on the importance sampling do not work

Challenges

Finite Density QCD

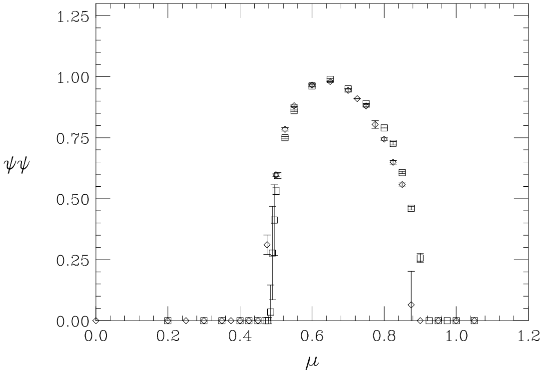
Two color QCD (SU(2)) gauge group: NO SIGN PROBLEM

Challenges

Finite Density QCD

Two color QCD (SU(2)) gauge group: NO SIGN PROBLEM

R. Aloisio, V. Azcoiti, G. Di Carlo, A. Galante, A.F. Grillo
Nucl. Phys. B606 322 (2001)



Challenges

Finite Density QCD

Three color QCD:

- Reweighting (restricted to small values of the chemical potential μ)
- Taylor series expansion (restricted to small values of the chemical potential μ)
- Analytical extensions

Challenges

Finite Density QCD

V. Azcoiti, G. Di Carlo, A. Galante, V. Laliena Nucl. Phys. B723 77 (2005)

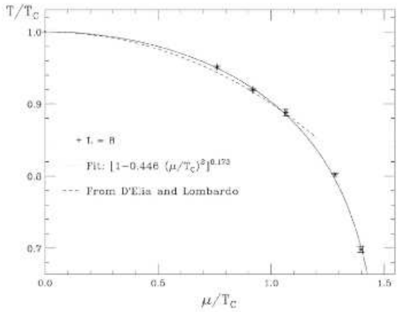


Fig. 8. Phase diagram in physical units, with the zero density transition temperature, T_C, setting the scale.

The best χ^2 fit gives $c \approx 0.446(9)$ and an exponent, $p \approx 0.173(7)$. One may be curious about the extrapolation of this line to zero temperature. If this were done, we would find

Challenges

θ-vacuum

- Since m_u and m_d are much smaller than the dynamical QCD scale Λ_{QCD}, the QCD Lagrangian has an approximate $U(2)_A$ symmetry. SCSB implies 4 near Nambu-Goldstone bosons. But $m_\eta \gg m_\pi$!
  The $U(1)_A$ anomaly solves the η problem but generates the Strong CP problem: extra-term in the QCD Lagrangian

$$\mathcal{L}_\theta = \frac{i\theta}{16\pi^2}\epsilon^{\mu\nu\alpha\beta} trF_{\mu\nu}F_{\alpha\beta}$$

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- In Condensed Mater Physics chains of half-integer quantum spins with antiferromagnetic interactions are related to the O(3) non-linear sigma model with topological term at $\theta=\pi$: Haldane conjecture

Challenges

θ-vacuum

First attempts to simulate Quantum Sytems with a Topological θ-term: flattening behavior in the vacuum energy density as an artifact of the simulation method.

- **G. Schierholz** Nucl. Phys. B (Proc. Suppl.) 42 270 (1995)
- **J.C. Plefka, S. Samuel** Phys. Rev. D56 44 (1997)
- **M. Imachi, S. Kanou, H. Yoneyama** Prog. Theor. Phys. 102 653 (1999)

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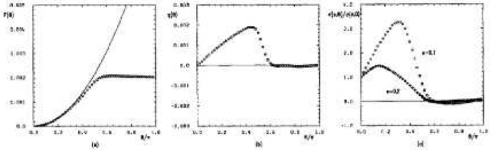

Figure 1. The free energy $F(\theta)$ (a), the charge density $q(\theta)$ (b) and the string tension $\sigma(\epsilon,\theta)$ (c) as a function of θ for the CP^3 model on the $V = 64^2$ lattice at $\beta = 2.7$. The solid curve in (a) is the prediction of the large-N expansion. Only the first half of the θ interval is displayed.

Challenges

θ-vacuum

- **V. Azcoiti, G. Di Carlo, A. Galante, V. Laliena** Phys. Rev. Lett. 89 141601 (2002)
- **V. Azcoiti, G. Di Carlo, A. Galante, V. Laliena** Phys. Lett. B563 117 (2003)

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written as $i\theta k_B T/2 \sum_i S_i$, where k_B is the Boltzmann constant and T is the temperature. For an even number of spins, the quantity $1/2 \sum_i S_i$ is an integer taking all values between $-N/2$ and $N/2$, and therefore it can be seen as a quantized charge. Furthermore, the theory has a Z_2 symmetry at $\theta = 0$ and $\theta = \pi$ which is analogous of CP in field theory. We use the notation $F = J/k_B T$, where J is the coupling constant between nearest neighbors. The transfer matrix technique allows one to solve analytically the model. For antiferromagnetic couplings, $F < 0$, the magnetization is an analytic function of θ between $-\pi$ and π. At $\theta = \pi$ the system shows a first order phase transition with a nonvanishing magnetization. From a numerical point of view the determination of the free energy density and order parameter through Eq. (1) in this model has the same level of complexity of more sophisticated models. Furthermore, in contrast to two-dimensional U(1) gauge theory, where the PDF of the topological charge is nearly Gaussian, the non-Gaussian behavior of the PDF of the mean magnetization in the antiferromagnetic Ising model makes this model a good laboratory to check the reliability of our approach. Figure 1 shows our numerical results for the order parameter versus θ for a linear chain of 1000 spins and $F = -1/2$. Statistical errors were estimated by doing ten samples of the numerical results and applying a jackknife analysis. The PDF of the order parameter for such a system takes values in a range of around 2000 orders of magnitude. Notwithstanding that, we are able to reproduce the order parameter in the whole θ interval within a few percent.

The two-dimensional compact U(1) gauge model with the θ angle at strong coupling constitutes another interesting check because we can compare the goodness of our approach with the other existing simulations which showed artificial behavior with a fictitious phase transition moving to the origin when increasing the lattice volume. Figure 2 displays our results for the topological charge density versus θ in a 80×80 lattice at $\beta = 0$ and $\beta = 0.6$. We are able to reproduce the exact result within a few per thousand in the whole θ interval. The agreement between analytical and numerical results is actually impressive. Furthermore, the flattening found in [3] for the free energy density in relatively small lattices is absent in our simulations even in the 80×80 lattice.

To test how different kinds of errors in the determination of the function $f(x)$ which defines the PDF of the density of topological charge can affect the determination of the free energy and order parameter, we have added to the measured $f(x)$ a random relative error of order 10^{-3}. Figure 3 shows the order parameter obtained in this way. As can be seen a small but random error in $f(x)$ propagates to the order parameter in a very dramatic way and makes the calculation meaningless. Contrary to that if, in order to simulate a correlated relative error of order up to 50%, we replace the measured $f(x)$ by the (even) function $f(x)(1 + 0.5\sin(x^2))$, the result for the order parameter is practically indistinguishable from the exact value for $\theta < \pi/2$, and the maximum deviation is about 25%, at $\theta = \pi$ (see Fig. 3). We conclude that random errors in $f(x)$ propagate in a very dramatic way but correlated errors do not, and this helps one to understand why our approach works so well.

The last model we have analyzed is CP^3 in two-dimensional Euclidean space. It is the standard wisdom that this model shares many qualitative features with QCD. Even if it has not been analytically solved we believe it is worthwhile to compare our results with previous existing numerical simulations. We studied the lattice formulation that makes use of an auxiliary U(1) field. Also in this model, the previous numerical simulations gave artificial phase transitions with a flattening

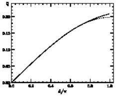

FIG. 1. Magnetization versus the imaginary magnetic field in the one-dimensional antiferromagnetic Ising model at $F = -1/2$ on a chain of 1000 sites: exact (dashed curve) and numerical (continuous curve) results. Statistical errors are smaller than 2%.

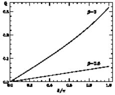

FIG. 2. Topological charge versus θ in the two-dimensional U(1) model at $\beta = 0$ and $\beta = 0.6$ on a 80×80 lattice. Statistical errors are not visible at this scale. The exact result (dashed curve) cannot be distinguished from the numerical result (continuous curve).

Challenges

θ-vacuum

- **V. Azcoiti, G. Di Carlo, A. Galante, V. Laliena** Phys. Rev. D69 056006 (2004)
- **V. Azcoiti, G. Di Carlo, A. Galante** Phys. Rev. Lett. 98 257203 (2007)
- **V. Azcoiti, G. Di Carlo, E. Follana, M. Giordano** Phys. Rev. D86 096009 (2012)

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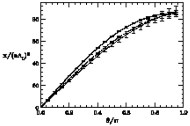
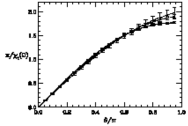


FIG. 3: Topological charge density divided by the squared lattice size (in units of Λ_L) versus θ/π for $\beta = 0.60, 0.65, 0.80$ (continuous, dashed, dotted line respectively) in the $L = 200$ lattice.



ues. In Fig. 3 violations to asymptotic scaling are at most ±6%. The origin of these violations could be the higher order corrections to the perturbative renormalization group equation and then, as seen in the plot, we would expect higher violations for the smaller β value. Indeed, as shown in Fig. 4, the non perturbative scaling is realized with much better accuracy: the three curves are now compatible within statistical errors. This is again a consistency check of both the results and the method(s).

To conclude we want to emphasize that the continuum θ dependence of a confining and asymptotically free quantum field theory has been fully reconstructed. Data collapse for different couplings within % level. The evidence for scaling in CP^9 model at non zero θ is the strongest indication that the CP symmetry is spontaneously broken in the continuum, as predicted by the large N expansion.

This work also shows how the inapplicability of the importance sampling technique to simulate physically interesting θ-vacuum models can be successfully overcome.

The authors thank the Consorzio Ricerca Gran Sasso that has provided the computer resources needed for this work. This work has been partially supported by an INFN-CICyT collaboration and MCYT (Spain), grant FPA2000-1252. Victor Laliena has been supported by Ministerio de Ciencia y Tecnologia (Spain) under the Ramón y Cajal program.

Conclusions and Outlook

- Lattice Field Theory is the most powerful non perturbative approach to Quantum Field Theory
- It is free from mathematical ambiguities: the path integral is well defined and the Theory is regularized to all orders in perturbation theory
- It has become a fundamental field in the High Energy Physics world
- Much progress has been reached during the 40 years of life of the field
- But still much to do
- Solve the two big challenges:
  - Finite density QCD
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Thank you for your attention