The 'Higgs' amplitude mode at the two-dimensional superfluid/Mott insulator transition

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Table of contents

Table of contents

Global transformations

Free theory for a complex scalar field Ψ:

$$\mathcal{L} = (\partial_\mu \Psi)^\dagger (\partial_\mu \Psi) + \mu^2 \Psi^\dagger \Psi - \lambda (\Psi^\dagger \Psi)^2$$

The Lagrangian is invariant under the global transformations group $U(1)$:

$$\Psi(x) \mapsto \Psi'(x) = e^{-i\alpha}\Psi(x), \quad \alpha \in \mathbb{R}$$

To include interactions, the global transformations are transformed into gauge (local) transformations.

Gauge transformations

The global transformations group is transformed into the gauge transformations group $U(1)$:

$$\Psi(x) \mapsto \Psi'(x) = e^{-i\alpha(x)}\Psi(x)$$

For the Lagrangian to be invariant, it is necessary to include a gauge vector field A_μ:

$$A_\mu(x) \mapsto A'_\mu(x) = A_\mu(x) - \frac{1}{g}\partial_\mu \alpha(x)$$

Gauge theory for a complex scalar field Ψ:

$$\mathcal{L} = (D_\mu \Psi)^\dagger (D_\mu \Psi) + \mu^2 \Psi^\dagger \Psi - \lambda(\Psi^\dagger \Psi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$D_\mu = \partial_\mu - igA_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

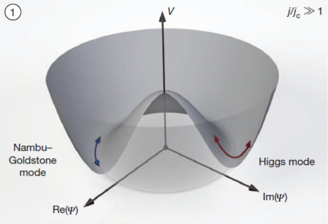
Vacuum expectation value

The potential in the Lagrangian is:

$$V(\Psi) = -\mu^2 \Psi^\dagger \Psi + \lambda(\Psi^\dagger \Psi)^2, \quad \mu^2 > 0$$

The vaccum expectation value is obtained at the minimum of the potential:

$$\langle 0|\Psi|0\rangle = \frac{1}{\sqrt{2}} v e^{i\varphi}, \quad v = \frac{\mu}{\sqrt{\lambda}}, \quad \varphi \in (-\pi, \pi]$$



There are infinite possible vacuums. The choice of a value for φ (e.g., $\varphi = 0$), causes a spontaneous breaking of the gauge symmetry $U(1)$.

Small oscillations from the minimum

The complex field Ψ can be written in polar coordinates.

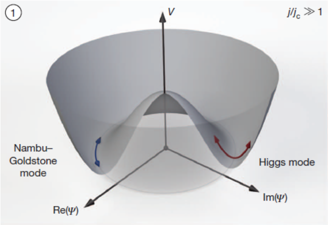
$$\Psi(x) = \frac{1}{\sqrt{2}}(v + \eta(x))e^{\frac{i\xi(x)}{v}}$$

For small oscillations:

$$\Psi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x) + \cdots)$$

The vacuum expectation values are:

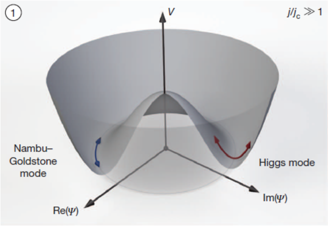
$$\langle 0|\eta|0\rangle = 0, \quad \langle 0|\xi|0\rangle = 0$$



The Nambu-Goldstone modes and the Higgs modes

Oscillations in the radial direction require energy to reach fields with higher potential. This implies the existence of massive bosons. The massive oscillations η are called Higgs modes.

Oscillations in the angular direction require no energy, because the new fields are again in the minimum of the potential. This implies the existence of massless bosons. The massless oscillations ξ are called Nambu-Goldstone modes.



Phase transition

If the value of μ^2 is reduced, the minimum of the potential moves closer to $\Psi = 0$. A phase transition occurs at $\mu^2 = 0$. When $\mu^2 < 0$, there is only one possible vacuum, so the gauge symmetry is not spontaneously broken.

- Superfluid phase: $\mu^2 > 0$, with spontaneous symmetry breaking.
- Mott insulator phase: $\mu^2 < 0$, without spontaneous symmetry breaking.

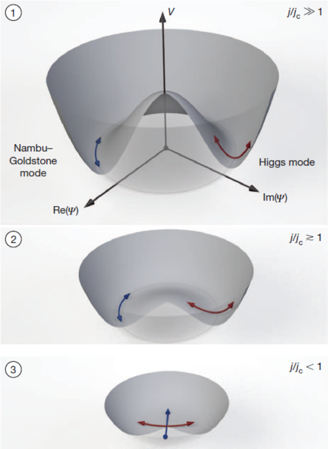


Table of contents

Motivation

The existence of Higgs modes is not clear in low dimensions. Whether such a mode exists in low-dimensional systems as a resonance-like feature, or it becomes overdamped through coupling to Nambu-Goldstone modes, has been a subject of debate (ref. 2-9).

Previous experiments:

- Unexpected peak in Raman scattering (ref. 10-11): earliest expermerimental evidence for a Higgs mode.
- Neutron scattering experiments on quantum antiferromagnets (ref. 12): mode spectrum across a quantum phase transition in a three-dimensional system.

Bose-Hubbar model

A model for a two-dimensional atom lattice is used:

$$H_{BH} = -J \sum_{<ij>} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1) + \sum_i (V_i - \mu) n_i .$$

Coupling parameter $j = \frac{J}{U}$: phase transition at j_c.

Theoretically, a modulation of the lattice depth can reveal a Higgs mode in 2 dimensions.

Experimental set-up

The experiment focus in ultracold bosonic atoms, set in an optical lattice. To study the possible existence of Higgs mode, a two-dimensional degenerate gas of ^{87}Rb atoms in a single antinode of an optical standing wave is used (ref. 23).

The lattice depth is modulated at variable frequencies. To quantify the response, the lattice depth is later increased to the non-interacting limit ($j \simeq 0$) and the temperature is measured.

Previous experiments used a lattice modulation amplitude of 20%. The high sensitivity of the measurement method allows to reduce the modulation amplitude to 3%.

Measurement process

Parameters: $\nu_{mod} = 1/\tau$, V_0, j/j_c.

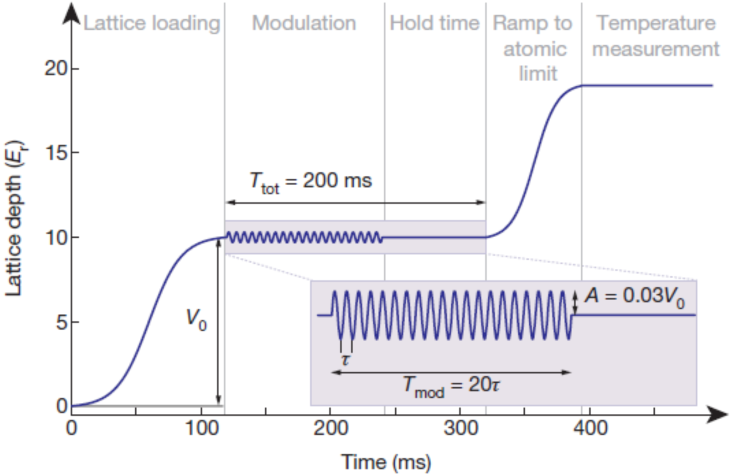
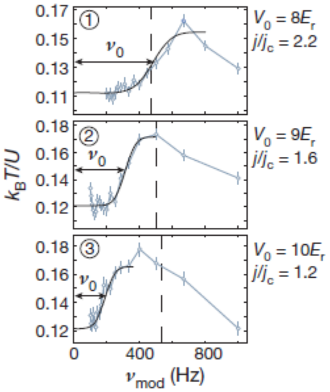


Table of contents

Temperature values

Fitting with an error function centered at ν_0:

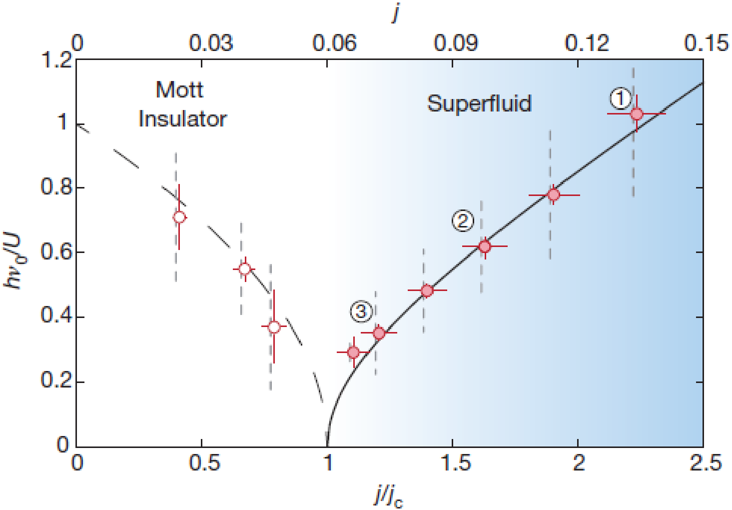
$$T(\nu_{mod}) = T_0 + \frac{\Delta T}{2}\left[\operatorname{erf}\left(\frac{1}{\sigma_e}(\nu_{mod} - \nu_0)\right) + 1\right]$$



Parameters:

- T_0: Initial value of the temperature.
- ΔT: Change in temperature.
- σ_e: Width of the ramp.
- ν_0: Frequency value at $T_0 + \frac{\Delta T}{2}$.

Softening of the Higgs mode



Softening of the Higgs mode

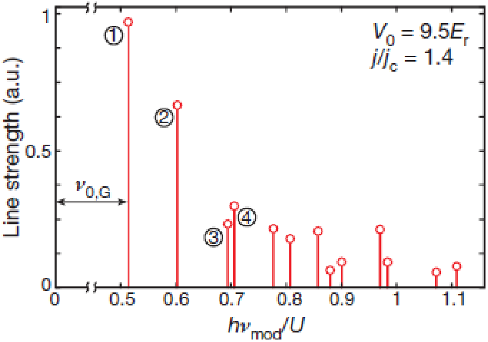
Superfluid phase ($j/j_c > 1$): Prediction for the Higgs gap at commensurate filling

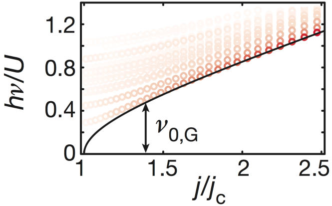
$$\frac{h\nu_{SF}}{U} = \sqrt{\left(3\sqrt{2}-4\right)\left(1+\frac{j}{j_c}\right)\left(\frac{j}{j_c}-1\right)}$$

Mott insulator phase ($j/j_c < 1$): Prediction by mean fiel theory

$$\frac{h\nu_{MI}}{U} = \sqrt{\left[1+\left(12\sqrt{2}-7\right)\frac{j}{j_c}\right]\left(1-\frac{j}{j_c}\right)}$$

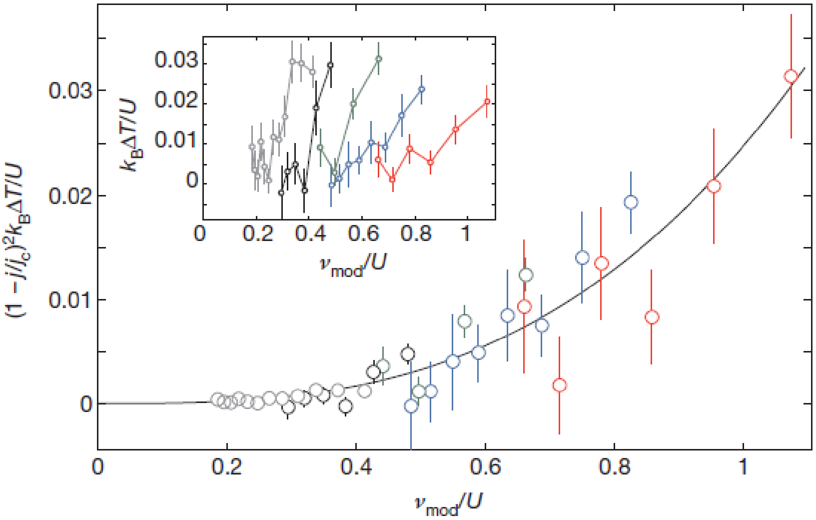
Eigenspectrum



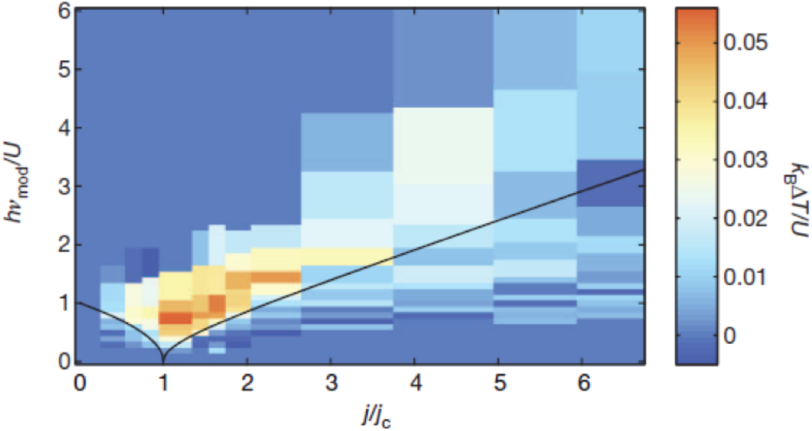


The gap frequency of the lowest lying mode follows the prediction for commensurate filling.

Low-frequency response



Weakly and strongly interacting limit - Experimental data



Weakly and strongly interacting limit - Simulation data

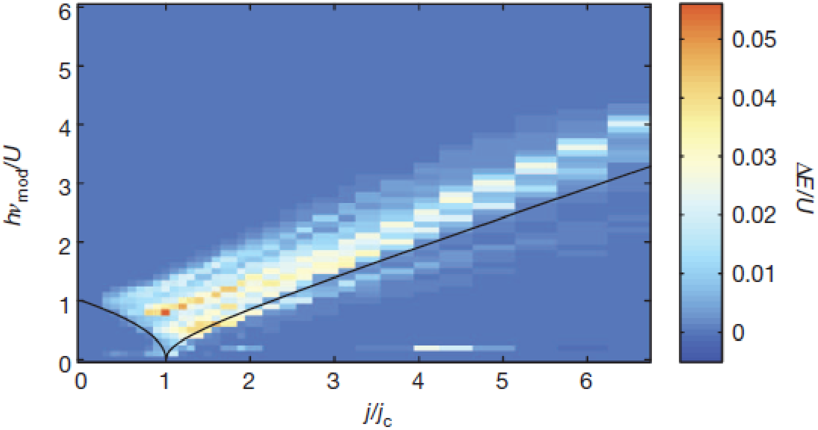


Table of contents

Conclusions

- Long-wavelength Higgs modes have been identified in a neutral, two-dimensional superfluid close to the quantum phase transition to a Mott insulator state.
- Recent advances in the high-resolution imagin of single atoms in optical lattices made possible to reach a new level of precision for the spectroscopy of ultracold quantum gases.
- The spectra show softening at the quantum phase transition, consistent with the generic ν^3 low-frequency scaling.
- The results require a quantitative theory capable of predicting the observed disappearance of the response. Also a study of the discrete nature of Higgs modes in a confined system is required.

Table of contents

Critics

- The experimental methods are clearly explained in the appendix and in the supplementary information. These sections also provide more experimental data and graphics and explain the analysis and calculations done in the article.
- Some of the important points are not made clear. It is not explained the relation between Higgs modes and temperature in the lattice, and how the experimental results led to the existence of these Higgs modes. This makes the article difficult to understand for those of us who are not expert in the subject.
- The motivation of the article is not well explained. Although some references are provided, only a few words are said about the possible non-existence of Higgs modes in low dimensions.