Causality in Quantum optics

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araid FUNDACIÓN AGENCIA ARAGONESA PARA LA INVESTIGACIÓN Y EL DESARROLLO

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Causality and the N-photon Scattering matrix in waveguide QED

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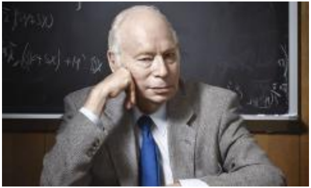
Soon in your arXiv mirrors! (we hope)

Brian Greene

Brian Greene

Brian Greene

Weinberg @ The Q. Theo. of Fields



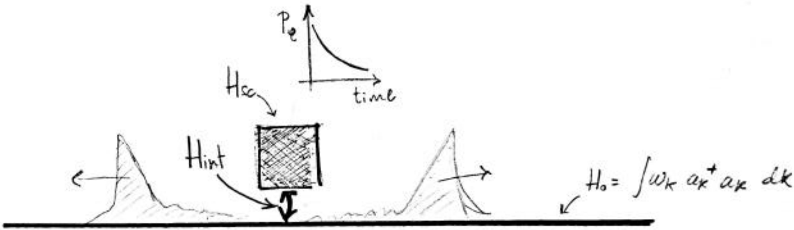
It is one of the fundamental **principles** of physics (indeed, of all science) that experiments that are sufficiently separated in space have unrelated results. The probabilities for various collisions measured at Fermilab should not depend on what sort of experiments are being done at CERN at the same time. If this principle were not valid, then **we could never make any predictions about any experiment without knowing everything about the universe**.

Where?

- Light-matter Hamiltonian ($\hbar = 1$),

$$H = \int dk\, \omega_k\, a_k^\dagger a_k + H_{sc} + \int dk\, (g_k G^\dagger a_k + g_k^* G a_k^\dagger).$$

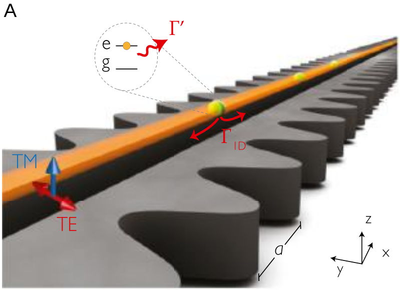
where $[a_k, a_{k'}^\dagger] = \delta(k - k')$.



- Non relativistic. Non solvable.

Waveguide & atoms

$$H = \int dk\, \omega_k\, a_k^\dagger a_k + H_{\rm sys} + \int dk\, (g_k G^\dagger a_k + g_k^* G a_k^\dagger).$$

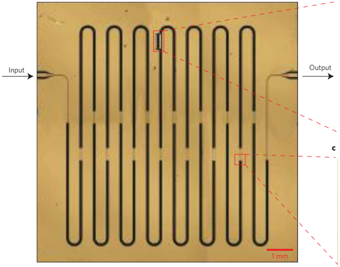


- Cs atoms @ alligator waveguide. Resonance freq ~ 350 THz

[Hood *et al*, PNAS 2016]

Superconductors

$$H = \int dk\, \omega_k\, a_k^\dagger a_k + H_{\mathrm{sys}} + \int dk\, \left(g_k G^\dagger a_k + g_k^* G a_k^\dagger\right).$$

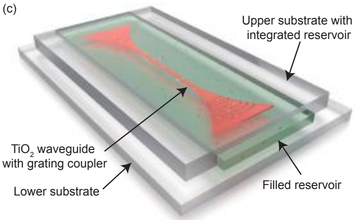


- Artificial atoms @ Transmission line. Microwaves

  [Liu *et al*, Nat Phys 2017, Forn-Díaz, Nat Phys 2017, ...]

Nanoguides (Today @ arXiv)

$$H = \int dk\, \omega_k\, a_k^\dagger a_k + H_{\mathrm{sys}} + \int dk\, \left(g_k G^\dagger a_k + g_k^* G a_k^\dagger\right).$$

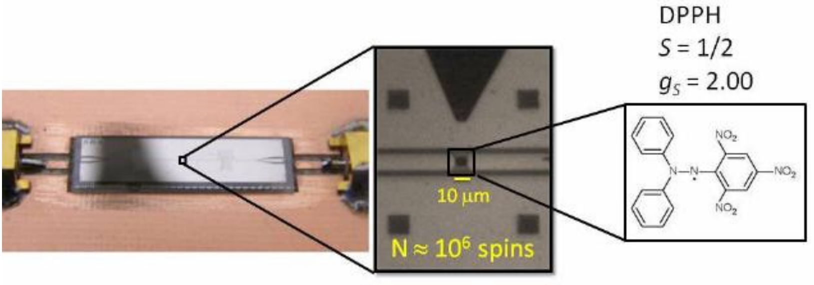


- Dye molecules @ nanoguide. Optical

[Türschmann *et al*, 1702.05923]

Hybrid @ Zaragoza

$$H = \int dk\, \omega_k\, a_k^\dagger a_k + H_{\mathrm{sys}} + \int dk\, (g_k G^\dagger a_k + g_k^* G a_k^\dagger).$$

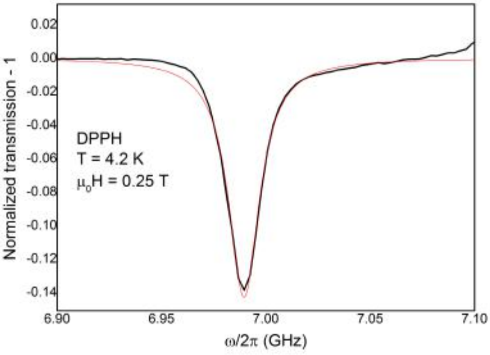


- Magnetic molecules @ Transmission line. Microwaves

[Exp: Fernando, Olivier, Mark, ...]

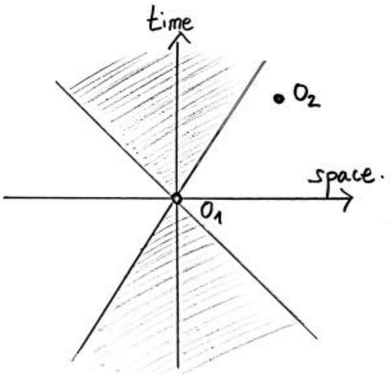
Hybrid @ Zaragoza

- Transmission (reflexion)



- Spectroscopy. Atom-atom interactions. Nonlinear optics @ few photon, ...

Causality in QFT

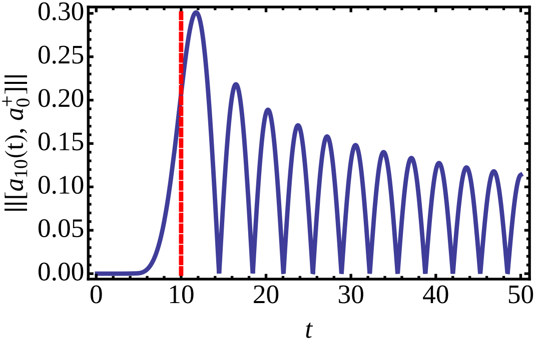


$$[O_1(x,t), O_2(y,t')] = 0 \qquad \forall\,(x,t),\ (y,t')\ \text{ s.t. }\ |x-y|-c|t-t'|>0\,,$$

[Tong Lectures @ Cambridge]

Example: non relativistic bosons, $[a_x, a_{x'}^\dagger] = \delta_{xx'}$

- Tight-Binding: $H = \sum_x \Omega a_x^\dagger a_x + J(a_x^\dagger a_{x+1} + \mathrm{h.c.})$
- Non relativistic $\omega(k) = \Omega + 2J\cos(k)$
- $v_k = d_k\omega(k) = -2J\sin(k)$.

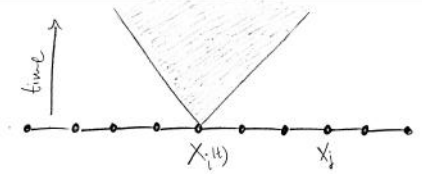


$(\max v_k = 2J = 1)$

Non relativistic → Lieb and Robinson

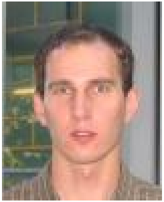
- The information propagates at finite velocity:

$$||[X_i(t), X_j]|| \leq c \; e^{-a(|i-j|-vt)}$$



[Lieb and Robinson, 1972]

Non relativistic $\rightarrow$ Lieb and Robinson $\rightarrow$ Causality

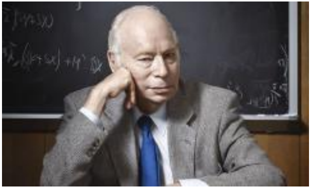


$\star$ Info exchanged is exponentially small in space-like separations.

[Bravyi, Hastings, Verstraete 2006]

- Other: Area laws, entanglement, simulations, correlation in gapped models, butterfly effect, this talk, ...

Name-dropping (part I)



... quantum mechanics plus Lorentz invariance plus cluster decomposition implies quantum field theory.

[Weinberg, 1997]

S-matrix

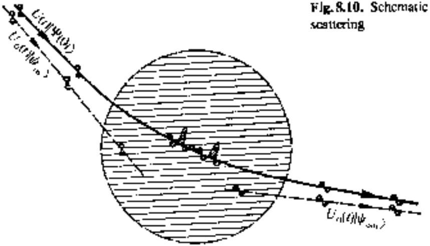


Fig. 8.10. Schematic description of quantum scattering

$$|\psi_{\text{out}}\rangle = S|\psi_{\text{in}}\rangle$$

[Galindo & Pascual, QM]

- Input $\to$ output states. They evolve freely (asymptotic cond.)

$$(S_{y_1\dots y_N x_1\dots x_N})_{\mu\nu} = \langle\Omega_\mu|a_{y_1}\dots a_{y_N}\lim_{t_\mp\to\mp\infty} U_I(t_-,t_+)a^\dagger_{x_1}\dots a^\dagger_{x_N}|\Omega_\nu\rangle$$

Cluster decomposition principle

- Laboratory 1: $\alpha_1 \to \beta_1$ / Laboratory 2: $\alpha_2 \to \beta_2$. If distant:

$$S_{\beta_1\ \beta_2,\alpha_1\ \alpha_2} \cong S_{\beta_1,\alpha_1} S_{\beta_2,\alpha_2}$$

- Always
$$S_{\beta_1\ \beta_2,\alpha_1\ \alpha_2} = S_{\beta_1,\alpha_1} S_{\beta_2,\alpha_2} + S^C_{\beta_1\ \beta_2,\alpha_1\ \alpha_2}$$

- Energy conservation (always) + translational Inv. (mom. cons)

$$\begin{aligned} S_{\beta_1,\alpha_1} &\sim \delta(k_1 - p_1) \\ S^C_{\beta_1\ \beta_2,\alpha_1\ \alpha_2} &\sim \delta(k_1 + k_2 - p_1 - p_2) + \mathrm{perm.} \end{aligned}$$

- Weinberg: Principle. Peskin: derived.

Causality, Cluster in waveguide QED?

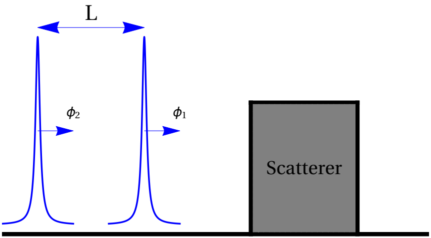
What we know about S (the model is not solvable)?

Wavepackets

- Localised wavepackets:

$$\psi_{\bar{k}\bar{x}}(t)^\dagger = \int e^{ik\bar{x}-i\omega_k t}\phi_{\bar{k}}(k)a_k^\dagger dk,$$

E.g. $\phi_{\bar{k}}(k) = \frac{1}{\sqrt[4]{2\pi}\sqrt{\sigma}}\exp\left[-(k-\bar{k})^2/4\sigma^2\right].$



- We need them! causality implies somehow localization

Free field causality

Theorem

Let the Hamiltonian $H_0 = \int dk\, \omega_k a_k^\dagger a_k$. *Assume that (i)* $|v_k| = |\partial_k \omega_k| \leq c$ *and well-behaved dispersion relation. Then,*

$$\|[\psi_{\bar{k}\bar{x}}(t), \psi_{\bar{p}\bar{y}}(t')^\dagger]\| = \mathcal{O}\left(\frac{1}{|d|^n}\right),\ d \to \infty.$$

for the wavepackets.

Proof.

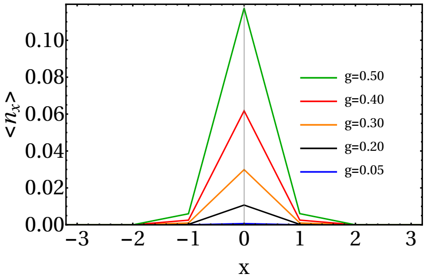
Heisenberg evolution gives the exponents $ik(\bar{x} - \bar{x}') - i\omega_k(t - t')$. They can be bounded by exponets as $ikz \to$. Riemann-Lebesgue: $\int \mathrm{e}^{ikz} f(k)\, dk \to 0$, as $z \to \infty$. Causality is linked to rapidly oscillations in the unitary dynamics that average to zero asymptotically. □

Scattering in light-matter (g.s)

Theorem

Far away the scatterer, the g.s. is trivial (the one of H_0).

$\langle \Omega_0 | O | \Omega_0 \rangle \simeq \langle \mathrm{vac} | O | \mathrm{vac} \rangle \to$



Proof.

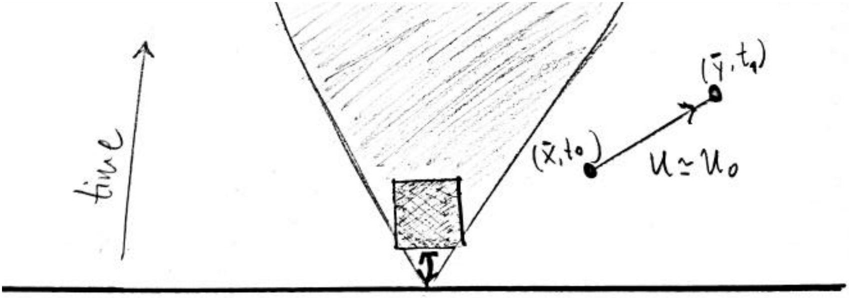
Energy fluctuations: $\langle \chi | \, H - E_0 \, | \chi \rangle = \langle O^\dagger [H, O] \rangle \geq 0$. $\to$ bound excitations $\to$ R-L lemma. $\square$

Scattering in light-matter (asymptotic)

Theorem

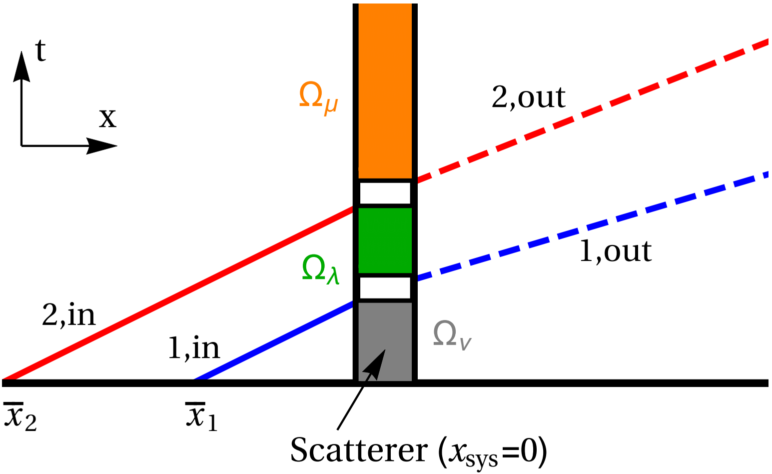
$$\psi_{\bar{k}\bar{x}}(t_1) \simeq U_0(t_1, t_0)^\dagger \psi_{\bar{k}\bar{x}}(t_0) U_0(t_1, t_0) + \mathcal{O}\left(\frac{1}{|d_{\min}|^{n-1}}\right),$$

Proof.



□

Cluster explained



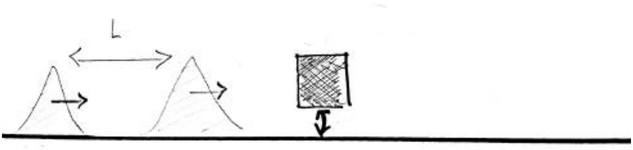
Cluster derived!

- Given:

$$A = \langle \Psi_{\rm out}|S|\Psi_{\rm in}\rangle = \langle \Omega_\nu|\psi_{\rm out} U(t_+, t_-)\psi_{\rm in}^\dagger|\Omega_\mu\rangle$$
$$= \langle \Omega_\nu|\psi_{\rm out}(t_+)\psi_{\rm in}(t_-)^\dagger|\Omega_\mu\rangle,$$

Theorem

Let us suppose the input state is



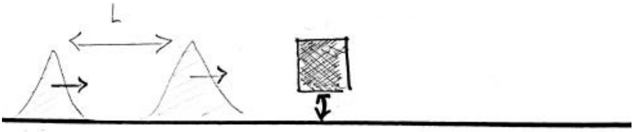
with $L \to \infty$ then,

$$A = \sum_\lambda A_{1,\nu\to\lambda} A_{2,\lambda\to\mu}.$$

Application 1

- Non solvable model (spin-boson type)

$$H = \epsilon \sum_x a_x^\dagger a_x - J \sum_x (a_x^\dagger a_{x+1} + a_{x+1}^\dagger a_x) + \Delta \sigma^+ \sigma^- + g(\sigma^- + \sigma^+)(a_0 + a_0^\dagger),$$



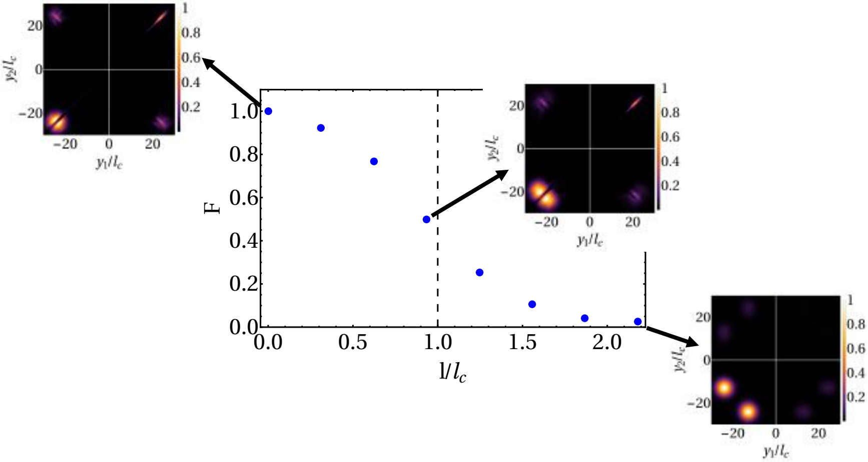
[Sanchez-Burillo, *et al* PRL 2014]

- Fluerescence $\sim$ Photon-photon interactions

$$F = \sum_{p_1, p_2} |\phi_{p_1,p_2}(t_+)|^2,$$

$$\omega_{p_1} + \omega_{p_2} = 2(\omega_{\bar{k}} \pm \sigma_\omega) \text{ and } \omega_{p_1}, \omega_{p_2} \notin (\omega_{\bar{k}} - \sigma_\omega, \omega_{\bar{k}} + \sigma_\omega).$$

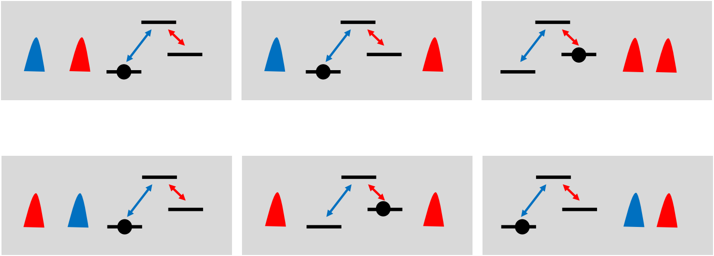
Application 1



Application 2

- Heaviside

$$(S^0_{y_1 y_2 x_1 x_2})_{\mu\nu} = \sum_{\lambda=0}^{M-1} (S_{y_1 x_1})_{\mu\lambda} (S_{y_2 x_2})_{\lambda\nu}\ \theta(y_2 - y_1) + [x_1 \leftrightarrow x_2, y_1 \leftrightarrow y_2]$$



- Elastic scattering: the order does not matter.

Application 2

- Inelastic $\rightarrow$ No single deltas in S^0 (Math: Heavisides)
- $S_0 \neq \delta(\omega_{k_1} - \omega_{p_1})$. In fact $F \neq 0$
- Localized wavepackets S_0 do not contribute to F.
- $F_T \sim \{e^{-\sigma L}, e^{-\gamma_T L}\}$ and $F_{S^0} \sim \{e^{-\sigma L}, e^{-\gamma_S L}\}$.
- $L \rightarrow \infty \rightarrow F = 0$.

Conclusions

1. Lieb-Robinson bounds $\rightarrow$ information outside the effective light-cone is useless. Causality.
2. Quantum optics: Causality derived. Causality $\rightarrow$ Cluster.
3. Subtleties @ inelastic scattering.

Bye

Muchas Gracias