Filiberto Ares
Departamento de Física Teórica
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Orthogonality Catastrophe

Martes Cuantico, April '17

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Orthogonality catastrophe:

$$\langle \mathrm{GS}' | \mathrm{GS} \rangle \to 0, \quad N \to \infty$$

Free Harmonic Oscillators

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N non-interacting Harmonic oscillators in 1D

$$H(\omega) = \frac{1}{2} \sum_{j=1}^{N} \left(-\partial_{x_j}^2 + \omega^2 x_j^2 \right)$$

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Ground state:

$$\Psi_\omega(\{x_j\}) = \prod_{j=1}^{N} \psi(x_j) = \left(\frac{\omega}{\pi} \right)^{N/4} \mathrm{e}^{-\frac{\omega}{4} \sum_{j=1}^{N} x_j^2}$$

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Take two different frequencies ω, ω'

$$\mathcal{A}_{\omega,\omega'} \equiv (\Psi_\omega, \Psi_{\omega'}) = \int \cdots \int \overline{\Psi_\omega(\{x_j\})} \Psi_{\omega'}(\{x_j\}) \prod_{i=1}^{N} \mathrm{d}x_i$$

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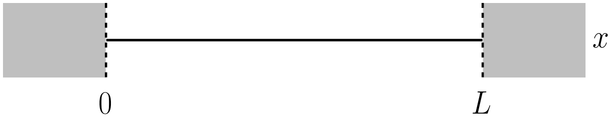
$$\boxed{\mathcal{A}_{\omega,\omega'} = \left[\frac{4\omega/\omega'}{(1+\omega/\omega')^2}\right]^{N/4} = \mathrm{e}^{-\xi N}; \quad \xi > 0}$$

Anderson's original OC

N non-interacting fermions in a 1D box of length L



P.W. Anderson PRL 18, 24 (1967)

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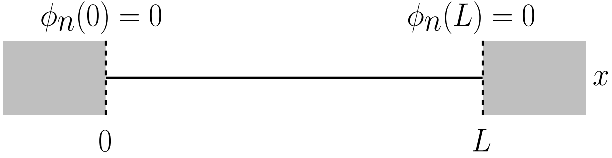

For each fermion

$$-\frac{\mathrm{d}^2\phi_n(x)}{\mathrm{d}x^2} = k_n^2\phi_n(x)$$

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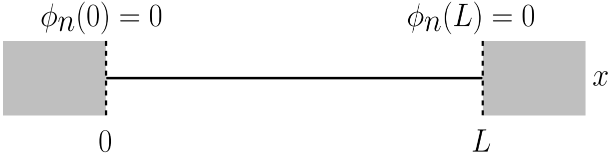

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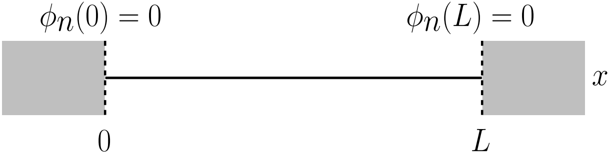

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N-fermion ground state ⇒ Slater determinant

$$\Phi_N = \frac{1}{\sqrt{N!}} \det\left(\phi_n(x_j)\right)_{n,j\leq N}$$

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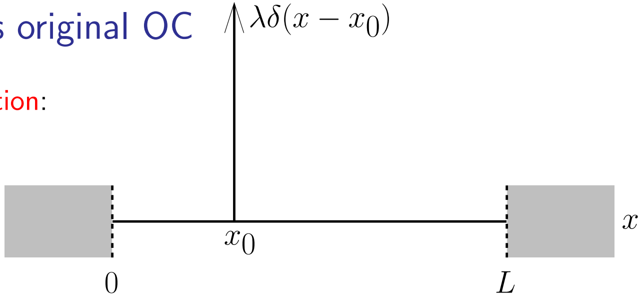

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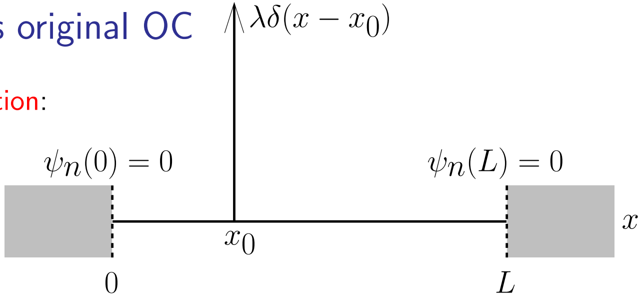

Now for a single fermion

$$-\frac{\mathrm{d}^2\psi_n(x)}{\mathrm{d}x^2} + \lambda\delta(x - x_0) = k'^2_n\psi_n(x)$$

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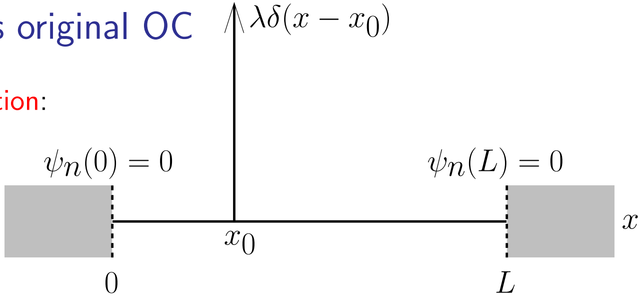

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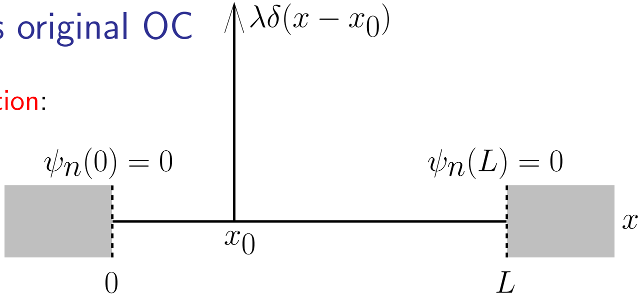

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Overlap of states before and after turn on $\lambda\delta(x - x_0)$

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Applying perturbation theory, if $\lambda \to 0$

$$k'_n \approx k_n + \delta_n/L; \quad \delta_n = \lambda \frac{\sin^2(k_n x_0)}{k_n}$$

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If $x_0 \ll L$

$$\mathcal{A}^{(1)}_{n,n'} \approx \frac{\pi n}{\pi(n + n') + \delta_{n'}} \frac{\sin \delta_{n'}}{\pi(n' - n) + \delta_{n'}}$$

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The N-particle overlap is bounded by the 1-particle overlaps

$$\mathcal{A} \leq \exp[-\sum_{\substack{n>N \\ n'<N}} |\mathcal{A}^{(1)}_{nn'}|^2]$$

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$\Rightarrow$ for large N, states close to the Fermi level dominate

$$\boxed{\mathcal{A} \leq N^{-(\sin \delta_F/\pi)^2}}$$

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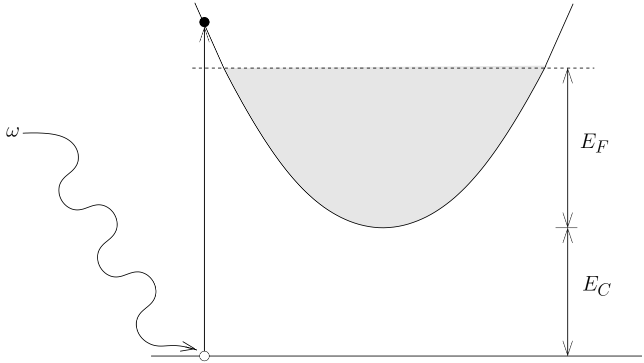
P.W. Anderson PRL 18, 24 (1967); PW Anderson, PR 164 (1967); P Nozieres, CT Dominicis, PR 178 1097 (1969)

Application I: X-ray absorption spectrum

Photon of frequency $\omega \geq E_F + E_C$ excites a core e^- of a metal to the conduction band



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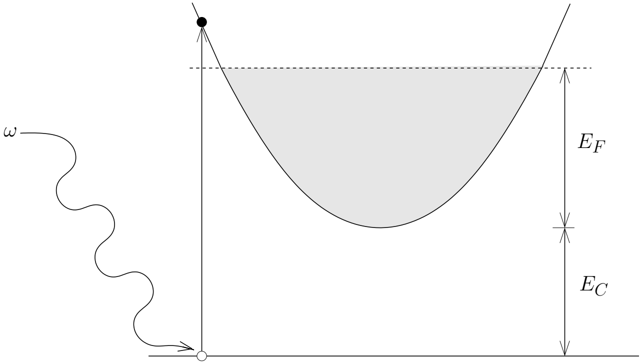

Interaction of the core hole with the Fermi sea: $\lambda\delta(x - x_0)$

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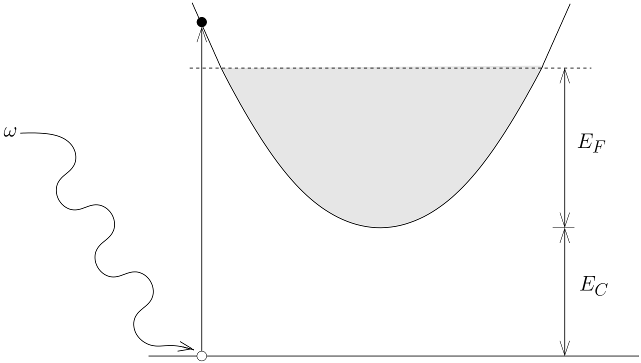

Interaction of the core hole with the Fermi sea: $\lambda\delta(x - x_0)$

$\Downarrow$

OC between states of the Fermi sea before and after the absorption!

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How does OC affect the absorption spectrum?

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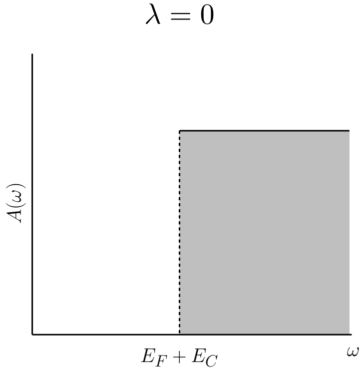

G D Manhan, *Many-Particle Physics*, 3rd Ed

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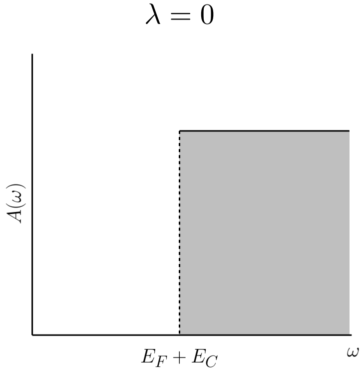

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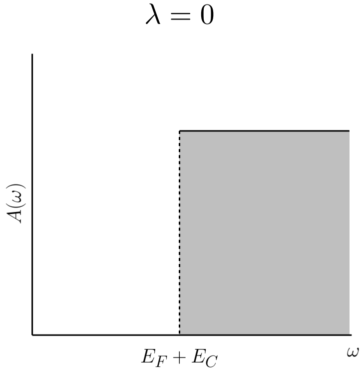

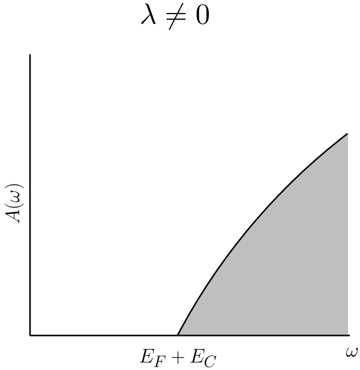

OC tends to suppress the X-ray absorption

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Application II: Kondo effect

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Scattering of conduction e^- in a metal due to magnetic impurities

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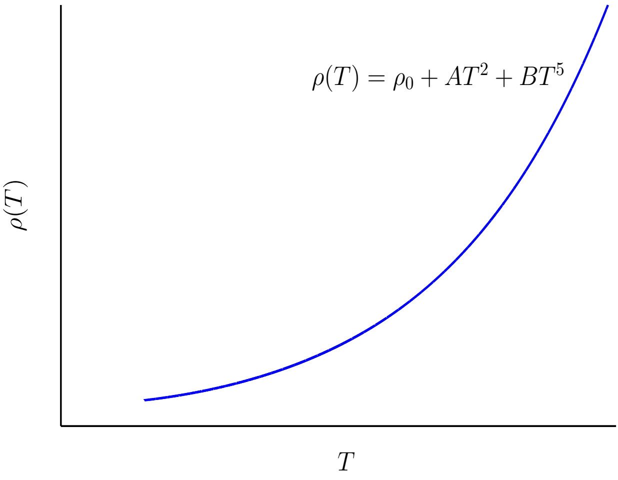
Scattering of conduction e^- in a metal due to magnetic impurities

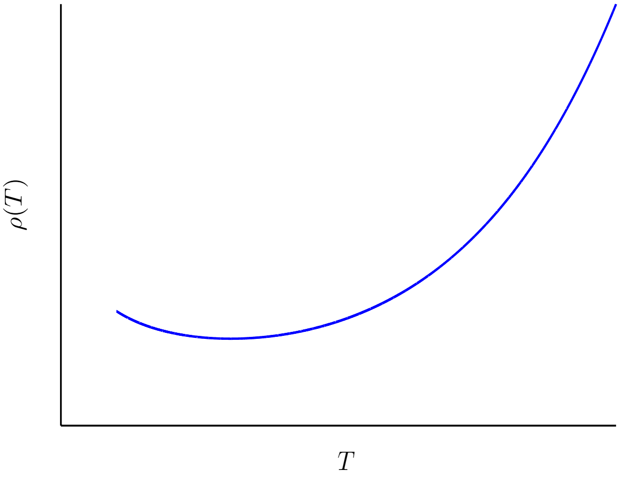
In general, electrical resistivity ρ of metals grows with temp. T



Application II: Kondo effect

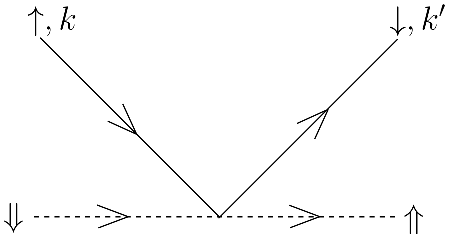
Scattering of conduction e^- in a metal due to magnetic impurities

In some cases, e.g. in gold, $\rho(T)$ shows a minimum



Application II: Kondo effect

$*$ Kondo: scattering e^-/magnetic impurity flipping their spins



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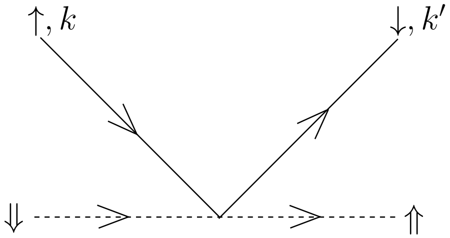

Perturbative computation $\Rightarrow$ logarithmic contribution to $\rho(T)$

$$\rho(T) = \rho_0 + AT^2 + BT^5 - C \log T$$

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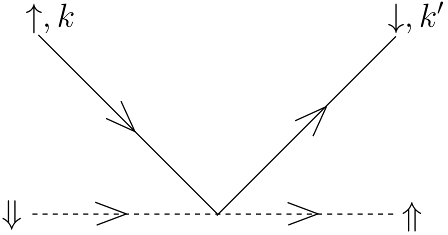

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Diverges when $T \to 0$ ⇒ Kondo problem!

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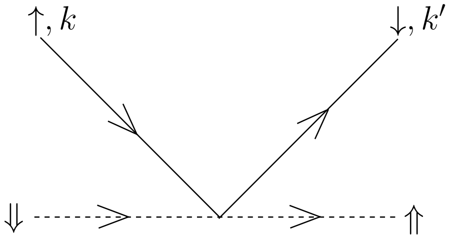

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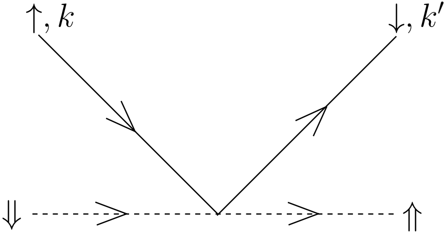
Below Kondo temperature: perturbative methods are not valid

Application II: Kondo effect

∗ Anderson: OC between states before and after a flipping



PW Anderson, PR 164 (1967)

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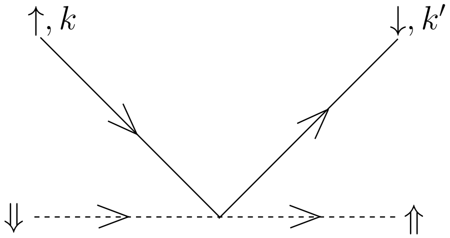

Coupling e^-/impurity leads to two competing effects with T:

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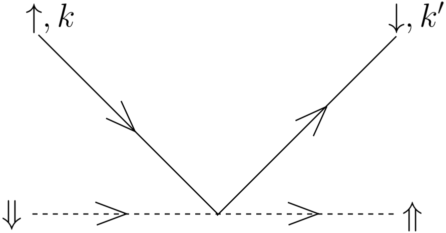

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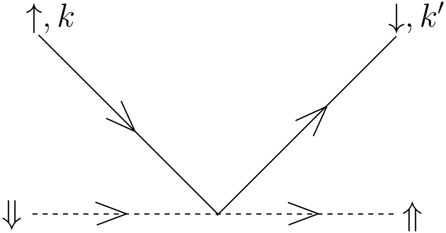

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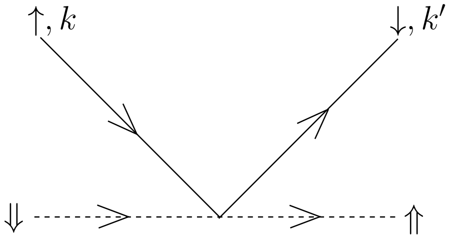

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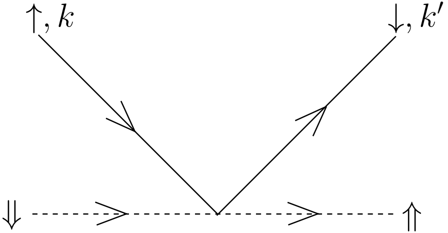

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} ⇒ Minimum of $\rho(t)$

Kondo temperature: scale at which spin-flips dominate over OC

PW Anderson, PR 164 (1967)

Sutherland model

B Sutherland, J Math Phys 12, 246 (1971)

Sutherland model

System of N particles in 1D described by

$$H(\omega,\mu) = \frac{1}{2}\sum_{j=1}^{N}\left(-\partial_{x_j}^2 + \omega^2 x_j^2\right) + \sum_{j=2}^{N}\sum_{k=1}^{j-1}\frac{\mu}{(x_j - x_k)^2}$$

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This system exhibits Haldane's exclusion statistics

B Sutherland, J Math Phys 12, 246 (1971)

What is Haldane's exclusion statistics?

$$H(\omega, \mu) = \frac{1}{2} \sum_{j=1}^{N} \left(-\partial_{x_j}^2 + \omega^2 x_j^2 \right) + \sum_{j=2}^{N} \sum_{k=1}^{j-1} \frac{\mu}{(x_j - x_k)^2}$$

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However, defining

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we have the spectrum of N free quasiparticles

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obeying the occupation rules

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OC in the Sutherland model

M Rajabpour & S Sotiriadis, PRA 89 (2014); FA, KS Gupta, AR de Queiroz, arXiv:1703.05363

OC in the Sutherland model

Take the ground state

$$\Psi_{\omega,\lambda}(\{x_j\}) = \mathcal{N}_{\omega,\lambda} \prod_{1\leq j<k\leq N} |x_j - x_k|^{\lambda} \mathrm{e}^{-\frac{\omega}{4}\sum_{j=1}^{N} x_j^2}$$

for two different sets (ω, λ), (ω', λ') and compute

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We have to solve a Selberg's integral...

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Selberg's integral: higher-dimensional generalization of Euler B function

$$\int_0^1 \cdots \int_0^1 \prod_{i=1}^{N} x_i^{\alpha-1}(1-x_i)^{\beta-1} \prod_{1\leq i,j\leq N} |x_i - x_j|^{2\gamma} \mathrm{d}x_1 \cdots \mathrm{d}x_N =$$

$$= \prod_{j=0}^{N-1} \frac{\Gamma(\alpha + j\gamma)\Gamma(\beta + j\gamma)\Gamma(1+(j+1)\gamma)}{\Gamma(\alpha+\beta+(N+j-1)\gamma)\Gamma(1+\gamma)}$$

$$\alpha, \beta, \gamma \in \mathbb{C}$$

$$\mathrm{Re}(\alpha),\ \mathrm{Re}(\beta) > 0$$

$$\mathrm{Re}(\gamma) > -\min\{1/N, \mathrm{Re}(\alpha)/(N-1), \mathrm{Re}(\beta)/(N-1)\}$$

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... after solving the Selberg's integral:

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Exponential OC with N^2

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$$\mathcal{A}_{\omega,\omega'} = \left[\frac{4\,{}^{\omega}\!/\!{}_{\omega'}}{(1 + {}^{\omega}\!/\!{}_{\omega'})^2}\right]^{N/4} = \mathrm{e}^{-\xi N}; \quad \xi > 0$$

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Sutherland model: free quasiparticles obeying exclusion statistics

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Exponential decay

- ∗ In localized disordered systems
  V Khemani *et al*, Nature Physics 11, 560 (2015)
  DL Deng *et al*, Phys Rev B 92, (2015)

¡Muchas gracias!